CHRISTER HENNIX

Notes on

TOPOSES & ADJOINTS

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NOTES ON
TOPOSES AND ADJOINTS

NOTHING IS MORE FANTASTIC, ULTIMATELY, THAN PRECISION.

ALAIN ROBBE-GRILLET

IN PHILOSOPHY WE ARE ALWAYS IN DANGER OF GIVING A MYTHOLOGY OF THE SYMBOLISM, OR OF PSYCHOLOGY: INSTEAD OF SIMPLY SAYING WHAT EVERYONE KNOWS AND MUST ADMIT.

LUDWIG WITTGENSTEIN

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NOTES FOR THE (NON-ALIEN) READER

NOTES FOR THE (NON-ALIEN) READER

1. ARROWS, I.E. LOGICAL CONNECTIONS, ARE INVISIBLE CONSTRUCTIONS MADE IN THE PRESENCE OF A LOGICAL SPACE OR L-SPACE.

THERE ARE INFINITELY MANY L-SPACES AVAILABLE, CORRESPONDING TO THE FORMS OF AWARENESS OR REALITIES THAT THE INTELLIGENCE MAY COMPREHEND.

THE INDICATION OF AN L-SPACE MUST NOT BE CONFUSED WITH ITS CORRESPONDING CONSTRUCTION. AN INDICATION IS JUST A SEMIOTICAL EVENT WHOSE SURFACE STRUCTURE REFLECTS SOME OR ALL OF THE ABSTRACTIONS UNDERLYING THE L-SPACE. THESE CONSTRUCTIONS OR SO-CALLED DEEP STRUCTURES OF A SEMIOTICAL EVENT ARE THE ACTUAL CONSCIOUSNESS BEING AWARE OF THE PRESENCE OF AN L-SPACE. IT IS ASSUMED THAT EACH SUCH PRESENCE IS REAL IF AND ONLY IF ITS MODAL SPECTRA HAVE ARRIVED FOR THE SITUATION IN QUESTION. MODALITIES, LIKE AWARENESS, COME IN DEGREES, THE DEGREE BEING SUFFICIENT OR NOT DEPENDING ON OPTATIVE CONTINGENCIES. MODALITIES CORRESPOND TO INTEGRATED FORMS OF CONSCIOUSNESS AND FORM THE SUBSTANCE OF THE AWARENESS OF THE UNIVERSE OF ALL LOGICALLY POSSIBLE WORLDS. WE THINK OF THIS UNIVERSE AS "STANDARD", I.E. AS A UNIVERSAL IN THE SENSE OF CATEGORY THEORY (FOLLOWING LAWVERE ET AL.).

2. THE PRESENT FORMAT OF OUR STYLE OF THINKING IS INSPIRED BY THE GREAT BREAKTHROUGHS IN THE STUDIES OF THE FOUNDATIONS OF MATHEMATICS. WE EMPHASIZE OUR AWARENESS OF THIS INTELLECTUAL TRADITION BY BRINGING FORTH TWO OF THE MOST CONTROVERSIAL CONTEMPORARY STARS IN THE FIELD TOGETHER WITH SOME OF THEIR MOST FAR-OUT CONCEPTUAL INNOVATIONS, MEANING OF COURSE ALEXANDER S.

YESENIN-VOLPIN'S ULTRAINTUITIONISTIC PROGRAM AND WILLIAM LAWVERE'S STUDIES OF MODELS OF TOPOI CONSTRUCTIONS.

3. THIS DEPENDENCY IS, HOWEVER, SUBORDINATED THE GENERAL PATTERN OF EPISTEMOLOGICAL ANARCHISM WHICH OUR COSMOLOGICAL CRITIQUE ENVELOPS. BESIDES SPOTTING EPISTEMOLOGICAL ILLUSIONS OUR CONCERN IS RELATED TO THE CREATION OF AN ATMOSPHERE OF ATTENTION IN EVERY (NON-ALIEN) SITUATION WHERE THE CONTRARY MAY OBTAIN. THAT IS, ONLY SUSTAINED FEELINGS OF AWARENESS ARE REAL OBJECTS IN OUR ONTOLOGY OF INTENSIONS. IT FOLLOWS THAT ANY SITUATION S WHICH IS NOT PARAMETERIZED BY Σ IS <u>ALIEN</u> FOR OUR PRESENT AIMS.

4. SPACES IN WHICH SUSTAINED FEELINGS OF AWARENESS OCCUR FORM A <u>PARTIAL ORDER</u> $<$ OF THEIR APPEARANCES. IN ABSENCE OF AN ATMOSPHERE OF ATTENTION THE <u>CONTINUITY PROPERTY</u> OF THIS ORDER IS VIOLATED. THE RESTORATION OF THE INJURED ORDER IS MARKED BY BARRING THIS ABSENCE (APPLICATION BAR - INDUCTION (BROUWER)).

5. THE ALIENNESS OF THE TACTICS OF ATTENTION BY WHICH THE ABOVE MODAL CONSIDERATIONS ARE SUSTAINED IS A FUNCTION OF THE <u>TOLERANCE OF ERROR</u> WHICH IS PERMITTED BY THE CORRESPONDING <u>OBJECTS OF ALIENATION</u>. IT IS SUGGESTED THAT EVERY SUCH CHARACTER IS EXPOSED FOR <u>DENIAL OF CONFIDENCE</u> SINCE EVERY OBJECT OF ALIENATION BY DEFINITION HAS ITS SPHERE OF CONFIDENCE REDUCED TO AN INFINITESIMAL. THIS BEING SAID, WE REMARK THAT THE CHOICE OF <u>BASIC CONSTRUCTION PRINCIPLES</u> IN <u>TOPOSES</u> AND <u>ADJOINTS</u> IS MADE WITH A PREFERENCE FOR THE LEAST PRESUMPTIOUS, ONLY TAKING

$\in$ (CANTORIAN MEMBERSHIP)

AND

$\sqcup$ (DIRECTED SUM)

AS UNDEFINED.

BY THESE PRIMITIVES, WE MAY RECOVER THE $\longrightarrow$ - OPERATION, VIZ. $\alpha \in \beta$ IF AND ONLY IF $\beta \longrightarrow \alpha$ AND $\sqcup\alpha \in \beta$ IF AND ONLY IF $\beta \longrightarrow \alpha_1$, OR $\beta \longrightarrow \alpha_2$ OR .... OR $\beta \longrightarrow \alpha_n$, FOR α_1, ..., $\alpha_n \in \alpha$

FOR EVERY PROCESS $\alpha_1 \rightarrow \alpha_2 \rightarrow$ .... $\rightarrow \alpha_k \rightarrow$ $\rightarrow \alpha_{k+1} \rightarrow$ .... THE FOLLOWING <u>DIALECTICAL TRIANGLE</u> IS <u>BASIC</u>, VIZ.

$$\begin{tikzcd}
\alpha_i = \alpha_j \arrow[rr] \arrow[dr] & & \alpha_j = \alpha_{j+1} \arrow[dl] \\
& \alpha_i \neq \alpha_{j+1} &
\end{tikzcd}$$

UNDERLYING THE INFINITELY PROCEEDING INVISIBLE PROCESSES PRESENTED IN TOPOSES AND ADJOINTS. (ALTHOUGH INFINITELY MANY IDENTIFICATIONS AND DISTINCTIONS ARE SUGGESTED BY THE <u>TIME CATEGORIES</u>, INITIAL SEGMENTS COVERED BY <u>GALOIS CONNECTIONS</u> SUFFICE FOR THE DESCRIPTION OF THESE <u>COHERENCE SINGULARITIES</u>).

6. IT GOES WITHOUT SAYING THAT THE PRESENT PRESENTATION ONLY REFLECTS THE FIELD FROM A RATHER SMALL (LIMIT) ORDINAL. ON THE OTHER HAND, BY FURTHER RARIFYING THE ATMOSPHERE OF ATTENTION FROM

A REFLECTION AT A HIGHER LIMIT ORDINAL, THE POSSIBILITY OPENS UP FOR A SUSTAINED FEELING OF AWARENESS EMBEDDED BY A CONTINUOUS REFLEXION PRINCIPLE (ALONG THE LINES OF BROUWER'S IDEAS OF THE CREATIVE SUBJECT, I.E. CONTINUOUS AT LIMITS). BUT THE USE OF THE REFLEXION PRINCIPLE AT LIMITS MUST BE GOVERNED BY SOME PRINCIPLE OF CAUTION, SINCE EMBARRASSING CONSEQUENCES MAY FOLLOW STEPS BEYOND PROJECTED LIMITS IF THE BASIC PRINCIPLES LACK THE PROPERTY OF BEING WELL-FOUNDED. THEREFORE, IN ALL EXTENSIONS OF THE NOTES ON TOPOSES AND ADJOINTS (I.E. SEMIOTICAL OBJECTS LISTED IN APPENDIX 3) THE PRESENCE OF COLLAPSING TECHNIQUES HAVE TAKEN PRECEDENCE. TOGETHER WITH AN ULTRA-INTIMATE PEDAGOGY GOVERNING THE INSTALLATION OF THE ENTIRE ENVIRONMENT TOPOSES AND ADJOINTS, THE GIVEN BASIC PRINCIPLES ARE INTENDED TO GROUND THE POSSIBILITY OF FOLLOWING A CONTINUOUS REFLEXION PRINCIPLE (LIKE FOLLOWING A CLEAR LANGUAGE $\mathcal{L}$).

7. TO THE ABOVE CLARIFYING TEXT WE WISH TO ADD SOME OF THE SOURCES OF OUR CONCEPTUAL FRAMEWORK.

L.E.J. BROUWER: COLLECTED WORKS I, ED. HEYTING, 1975

L.E.J. BROUWER: COLLECTED WORKS II ED. TROELSTRA, 1976

A.S. YESENIN-VOLPIN: INTUITIONISM AND PROOF THEORY, ED. KINO, MYHILL, VESLEY, 1970.

W. LAWVERE: SPRINGER LECTURE NOTES IN MATHEMATICS NO. 445. 1975.

SOME READERS MAY ALSO ENJOY

C. HENNIX: BROUWER'S LATTICE (MODERNA MUSEET, 1976).

FINALLY, WE MUST EMPHASIZE THAT THE FOLLOWING TEXT ONLY CONTAINS RATHER CONDENSED EXCERPTS FROM NOTEBOOKS THAT WE HAVE WRITTEN DURING THE LAST FIVE YEARS OR SO.

SPECTRA OF MODALITIES

AND

THE THEORY OF

THE CREATIVE SUBJECT Th(Σ)

SPECTRA OF MODALITIES
AND
THE THEORY OF THE CREATIVE SUBJECT, $T_H(\Sigma)$

I. IT IS A WELL-KNOWN FACT THAT ACTIVITIES ENDING IN ART MAY NOT ALWAYS CARRY A WELL-DETERMINED MEANING OR SENSE. ON THE CONTRARY, THE LACK OF MEANING(-FULNESS) IS COMPENSATED FOR BY (PURPORTED) AIM(S) EXPRESSED BY THE PURPOSE(S) GOVERNING THE INSTALLMENT OF THE GENERATING ACTIVITIES FOR WHICH END SOME PARTICULAR OBJECT IS TAKEN AS A WITNESS. THIS SOMEWHAT CONFUSED REALITY SHOWS THE INDISPUTABLE IMPORTANCE OF THE INTERPRETATION OF THE <u>OPTATIVE MODALITIES</u> UNDERLYING ANY GOAL-ORIENTED ACTIVITY $\mathcal{A}$.

II. IN ORDER TO FIX THE PURPOSEFULNESS OF AN ACTIVITY $\mathcal{A}$, SEVERAL RATIOS ARE TO BE MEASURED, SUCH AS

1) $$\frac{\text{INTEREST OF RESULTS}}{\text{EFFORTS INVOLVED}}$$

AND

2) $$\frac{\text{LONG-TERM SATISFACTION}}{\text{EFFORTS REQUIRED}}$$

GIVEN SOME SATISFACTORY MEASURES OF THESE RATIOS FOR AN ACTIVITY $\mathcal{A}$ THERE IS A FURTHER REQUIREMENT ON THE <u>MEANS</u> AVAILABLE BY WHICH THE RESULT(S) OF $\mathcal{A}$ ARE ACHIEVED. VIZ. IT IS GENERALLY REQUIRED THAT ANY AIM ACHIEVED THROUGH $\mathcal{A}$ IS ACQUIRED ONLY AS FAR AS <u>FAIR</u> MEANS HAVE BEEN PROVIDED.

Any violation of this fairness principle is to be considered harmful for the continuation of the situations S generated by $\mathcal{A}$, on account of the displacement of modalities, caused, in particular, by the displacement of goals relative the installment of $\mathcal{A}$.

III. Clearly, the condition of fairness for purposeful activities $\mathcal{A}$ imposes an obvious restriction as to the availability of means for realizing $\mathcal{A}$. This restriction must be evaluated relative the higher-order aims under which $\mathcal{A}$ is subsumed. As far as clarity and certainty is sought, the fairness principle is but a higher-order means for our epistemic development, and its violation poses obstacles as far as (foundational) communication is aimed at (to wit, its deepest threat).

IV. For the purpose of restricting the above-mentioned restriction, two spectra of modalities are defined for the sake of optimal freedom in activities $\mathcal{A}$ restricted by fair means. Viz.

(1) The first spectrum which will be designated Freedom$_1$ or F_1 and is defined as that regime $\mathbf{P}$ governing activities $\mathcal{A}$ in the absence of any obstructions. Id est, F_1 assigns the following interpretation to the fulfilment of the optative modalities connected with $\mathcal{A}$: If T is an aim in $\mathcal{A}$ and $\mathcal{A}$ provides (α) sufficient means and (β) all necessary means for realizing T in $\mathcal{A}$, then T is fulfillable in $\mathcal{A}$. F_1 clearly corresponds to the purposefulness of $\mathcal{A}$ and would be violated in any situation where (α) (β) hold but T has been (or will be) obstructed.

(2) The second spectrum which will be designated Freedom$_2$ or F_2 and which is defined as that regime $\mathbf{P}$ governing activities $\mathcal{A}$ such that no act in $\mathcal{A}$ is forced by coercion, fraud or any other violation of the fairness of means provided for the course of $\mathcal{A}$. Id est, $\mathcal{A}$ is said to possess property F_2 whenever every act in $\mathcal{A}$ is free from compulsion, id est exercised in accordance with the creative subject's free will. Clearly, the satisfiability of F_2 captures exactly what is intended by justfulness of $\mathcal{A}$ and the spectrum of F_2-modalities is precisely all instances of installments of given activities $\mathcal{A}$ for which fair means have been provided.

V. The spectra of modalities for $\mathcal{A}$ thus split into two basic components, F_1 and F_2. By passing to the direct limit of the projections of F_1 and F_2 on any $\mathcal{A}$, the composite $F_1\ F_2$ obtains. It corresponds to that regime $\mathbf{P}$ for which Freedom$_1$ and Freedom$_2$ hold simultaneously and if $\mathcal{A}$ possesses property $F_1\ F_2$ we shall say that $\mathbf{P}$ is eleutheric for $\mathcal{A}$ or that the activities comprised by the situations generated by $\mathcal{A}$ are eleutheric activities in the regime $\mathbf{P}$.
- The basic tenet of Yesenin-Volpin's Eleutheric Ethics or our Ultra Ethics is that all modalities connected with just and purposeful activities $\mathcal{A}$, including their spectra, are reducible to the modalities of the direct limit $F_1\ F_2$.

VI. A rough indication of the growth of $\mathcal{A}$ may be given by describing the tree structure associated with $\mathcal{A}$, designated by $\mathcal{T}_{\mathcal{A}}$. At its root, $\langle\quad\rangle_{\mathcal{S}}$, $\mathcal{T}_{\mathcal{A}}$ indicates the initial act in $\mathcal{A}$, while at every branch $\langle\ldots\ldots\rangle_{\mathcal{S}}$ above $\langle\quad\rangle_{\mathcal{S}}$, $\mathcal{T}_{\mathcal{A}}$ indicates for each node of a branch, the category of the decision invited at the node in addition

TO THE INTENSION FOR THAT CATEGORY AT THE GIVEN LOCI.

TREES OF THE KIND $\mathfrak{T}_{\mathcal{A}}$ DEPICT THE SEARCH IN WHICH THE CREATIVE SUBJECT DEVELOPS ACTIVITIES FOR THE PURPOSE OF ATTAINING SOME DESIRED RESULT(S) INVOLVING $\mathcal{A}$.

TO DESCRIBE THE INNER MAP PRESUPPOSED BY WAY OF $\mathfrak{T}_{\mathcal{A}}$, IT IS NOT SUFFICIENT TO LOOK ONLY AT THE INDICES AT THE NODES OF $\mathfrak{T}_{\mathcal{A}}$ BUT, IN ADDITION, IT IS NECESSARY TO MAP $\mathfrak{T}_{\mathcal{A}}$ INTO AN EXTENDED TREE, $\mathfrak{T}_{\Sigma}$, IN WHICH THE SEARCH IN $\mathcal{A}$, FUNCTOR-WISE, GOES OVER INTO THE DOMAIN OF THE THEORETICAL ACTIVITY CONNECTED WITH THE FULFILMENT OF THE SEARCH IN $\mathcal{A}$. THIS FUNCTOR IS CALLED THE $\Sigma\mathfrak{T}$ -PROJECTUM OF TREES OF KIND $\mathfrak{T}_{\mathcal{A}}$ (CF. DIALOGUE AND IMPASSE DIAGRAMS).

VII. WHEN UNDERSTOOD AS AN EPISTEMIC OPERATOR, THE CREATIVE SUBJECT, Σ , MAY BE CONCEIVED OF AS GROUNDED BY THE FOLLOWING AXIOMS. THE CLOSURE OF THESE AXIOMS UNDER THE CONSEQUENCE RELATION IS DENOTED TH(Σ), I.E. THE THEORY OF Σ.

AXIOMS FOR Σ :

I: $\Sigma \vdash_n \mathscr{A} \vee \Sigma \vdash_n \neg\mathscr{A}$

II: $\Sigma \vdash_n \mathscr{A} \rightarrow \mathscr{A}$

III: $\Sigma \vdash_n \mathscr{A} \ \& \ n > m \rightarrow \Sigma \vdash_m \mathscr{A}$

IV: $\neg\exists(x)\, \Sigma \vdash_x \mathscr{A} \rightarrow \neg\mathscr{A}$

V: $$\frac{\Sigma \vdash_n \mathscr{A}, \quad \mathscr{A} \rightarrow \mathscr{C}}{\Sigma \vdash_n \mathscr{C}}$$

$$\text{VI}: \frac{\Sigma \vdash_n \exists(x)\, \mathscr{A}(x)}{\exists(x)\ \Sigma \vdash_n \mathscr{A}(x)}$$

INFERENCE SCHEMA FOR $\mathrm{T}_H(\Sigma)$

$$\text{VII}: \frac{\Sigma \vdash_n \mathscr{F}\ ,\quad \Sigma \vdash_m \mathscr{F} \rightarrow \mathscr{G}}{\Sigma \vdash_{n,m} \mathscr{G}}$$

THE GENERAL IDEA IS NOW AS FOLLOWS. THE BASIC RELATION $\vdash$ IN THE CONTEXT $\Sigma \vdash_n \mathscr{A}$ IS INTERPRETED AS " Σ HAS DECIDED OR SOLVED $\mathscr{A}$ AT THE N^{TH} STAGE OF HIS INVESTIGATION OR RESEARCH IN $\mathcal{A}$", WHERE $\mathscr{A}$ DESIGNATES AN INTENSION OR PROBLEM CONNECTED WITH $\mathcal{A}$. THE LOGICAL OPERATORS $\neg$, & , $\mathbf{V}$ AND $\rightarrow$ ARE GIVEN THE FOLLOWING INTERPRETATIONS.

$\neg\ [\mathscr{A}]$ SIGNIFIES THE TASK OF OBTAINING THE ABSURDITY OF THE SOLUTION OF $\mathscr{A}$

& $[\mathscr{A}, \mathscr{B}]$ " " " " SOLVING BOTH $\mathscr{A}$ AND $\mathscr{B}$

$\mathbf{V}\ [\mathscr{A}, \mathscr{B}]$ " " " " " EITHER $\mathscr{A}$ OR $\mathscr{B}$

$\rightarrow [\mathscr{A}, \mathscr{B}]$ " " " " " THE PROBLEM $\mathscr{B}$ GIVEN ANY SOLUTION OF $\mathscr{A}$

IN ADDITION:

$\exists(x)\left[\,\cdot\,\mathscr{A}(x)\right]$ SIGNIFIES THE TASK OF SOLVING THE VALUE OF x SUCH THAT $\mathscr{A}(x)$;

WHERE $\exists(x)$ IS THE OPERATOR DESIGNATING THE <u>EXISTENTIAL CONSTRUCTION PRINCIPLE</u>.

AMONG SOLVABLE PROBLEMS IN TH(Σ) WE MENTION THE FOLLOWING TASKS (EXERCISE!):

$$\mathscr{A} \to \mathscr{B} \;\&\; \mathscr{B} \to \mathscr{C} \to \mathscr{A} \to \mathscr{C} \qquad (1)$$

$$\mathscr{A} \to \mathscr{C} \;\&\; \mathscr{B} \to \mathscr{C} \to \mathscr{A} \vee \quad \to \mathscr{C} \qquad (2)$$

$$\neg\mathscr{A} \to \mathscr{A} \to \mathscr{B} \qquad (3)$$

$$\mathscr{A} \to \mathscr{B} \to \neg\mathscr{B} \to \neg\mathscr{A} \qquad (4)$$

$$\mathscr{A} \to \neg\mathscr{B} \to \mathscr{B} \to \neg\mathscr{A} \qquad (5)$$

$$\neg(\mathscr{A} \vee \mathscr{C}) \to \neg\mathscr{A} \;\&\; \neg\mathscr{C} \qquad (6)$$

$$\neg(\mathscr{C} \;\&\; \mathscr{E}) \;\&\; \mathscr{C} \vee \neg\mathscr{C} \to \neg\mathscr{C} \vee \neg\mathscr{E} \qquad (7)$$

$$\neg(\mathscr{A} \;\&\; \neg\mathscr{A}) \qquad (8)$$

<u>REMARK</u>: BY (8), <u>CONSISTENCY</u> OF TH(Σ) IS ESTABLISHED, I.E. TH(Σ) IS A <u>NON-TRIVIAL</u> THEORY (I.E. EPISTEMICALLY PALATABLE).

<u>(HISTORICAL) REMARK</u>: <u>L.E.J. BROUWER</u> WAS THE FIRST LOGICIAN TO FORMULATE PRINCIPLES SIMILAR TO THE ONES GIVEN ABOVE.

THE HIERARCHY OF DEGREES OF MODALITIES

$\vdots$		
(n°)		INCREASINGLY STRICTER AND STRICTER CONDENSATIONS OF ACTS OF CONFIDENCE (BY ARROWS)
(5°)	$\mathcal{A} \rightarrowtail \Sigma^{\Sigma}$	ELEUTHERIA (IN ARROWS)
(4°)	$\not\rightarrowtail$	METAMATHEMATICS (CONSISTENCY OF ARROWS (HILBERT'S 2ND PROBLEM))
(3°)	$\underset{\sim}{\Sigma}$	ACTIONS AND PROCESSES (WITH ARROWS)
(2°)	Σ	AWARENESS (OF ARROWS)
(1°)	[ ]	ARROWS (AT POINTS OF CONDENSATION)
(0°)	$\square$	PLASMA (PER ARROWS)

VIII: Let us now briefly pause over the formalism so far introduced for the purpose of achieving a broader view of our subject.

It is a general fact that any activity $\mathcal{A}$ proceeds on the background of other activities $\mathcal{A}_i$ for which the relation

$$(*) \quad \mathcal{A} \sqsubset \mathcal{A}_i$$

holds for every i (where $\mathcal{A} \sqsubset \mathcal{A}_i$ reads $\mathcal{A}$ is included in $\mathcal{A}_i$ or $\mathcal{A}_i$ extends $\mathcal{A}$).

The exteriorized elements of $\mathcal{A}$, i.e. hose belonging to some $\mathcal{A}_i$ but not in $\mathcal{A}$, can often be encountered as objects mentioned in any relevant part of $\mathcal{A}$. But, significantly, they may not participate constructively in any creative situation in which $\mathcal{A}$ is realized (as opposed to the given interior elements of $\mathcal{A}$). The processes of exteriorizing parts of an activity that is given, proceed on the unfolding of the modal components of the activity, which, essentially, determine the scope or ranges of the conceptual frameworks that go into the interior of the activity.

When we say that a modal component acts on an activity, we are referring to a hierarchy of (degrees of) modalities, that, when fully developed, constitutes the spectra of modalities in $\mathcal{A}$ (cf. the diagram of the hierarchy of degrees of modalities).

By a rank 2 or 2nd degree modal operator is to be understood those modalities which participate in shorter (i.e. fragmentary) lines of reasonings or other more general activities involving the use of interpreted signs (like the bracketing operation). Rank 1 is assigned to those modalities

WHOSE CARRIER ACTIVITIES CONSIST IN THE USE OF (AT MOST) UNINTERPRETED SIGNS OR NO SIGNS AT ALL (LIKE THE SINGULAR AWARENESS OF THE PRESENCE OF A PLASMA). THERE IS NO UPPER BOUND ON THE ASSIGNMENT OF RANKS TO MODALITIES IN THIS HIERARCHY, AS SHOULD BE EXPECTED IF $\mathcal{A}$ HAS THE PROPERTY F_1 F_2. THE RANK OF A MODALITY REFLECTS THE COMPLEXITY OF ITS DEPENDENCE ON MODALITIES OF LOWER RANK AS WELL AS OF THE DEGREE OF CONFIDENCE VINDICATED BY THE SITUATIONS GENERATED BY $\mathcal{A}$. ANY $\mathcal{A}$ FOR WHICH THE PARTICIPATING MODALITIES ARE ASSIGNED RANK $\geqslant 3$ IS CALLED A THEORETICAL ACTIVITY. ON THE OTHER HAND, RANKS BELOW 3, I.E. < 3, ARE CONNECTED WITH DREAM ACTIVITIES. (THERE EXIST SOME BORDERLINE CASES OF 3RD DEGREE MODALITIES, NOTABLY INTUITIONISTIC MODAL MUSIC, WHICH, ALTHOUGH GROUNDED IN THE LAW OF SUFFICIENT REASON, NEVERTHELESS MAY BE SAID TO BE A DREAM ACTIVITY, DUE TO THE PRESENCE OF A DEONTIC MIRACLE IN THE ACTIVITY).

AS TO BE EXPECTED, THE THEORETICAL ACTIVITY OF Σ, AS DETERMINED BY THE $\Sigma\mathcal{T}$-PROJECTUM, MAKES ESSENTIAL USE OF MODALITIES OF RANK > 3. IN PARTICULAR, FOR ANY PROSPECTIVE INTERPRETATION OF AN ELEMENT IN THE CLASS OF OBJECTS REFERRED TO AS TOPOSES, IT IS ESSENTIAL TO NOTE THE RANK OF MODALITIES PARTICIPATING IN THE OBJECT. BY WAY OF EXAMPLE (AND INTIMATE PEDAGOGY!), IN TOPOSES RANK IS INCREASED BY INTRODUCING STRONGER CONCEPTS OF INFINITY AT POINTS WHERE AN END COULD HAVE BEEN PRESCRIBED. THE UN-ENDING OF A PROCESS, I.E. ITS POINTS OF INFINITY, IS CONNECTED WITH THE FUTURE OF ITS UNFOLDING, WHICH MAY BE NEAR OR REMOTE. BY REASONINGS ABOUT THE SCALES BETWEEN THESE POINTS, WE CAN IMAGINE DIFFERENT LENGTHS OF RAMIFYING INFINITARY PROCESSES INCLUDING THEIR "ZEONIAN" EMBEDDINGS WHICH DISTINGUISH THEMSELVES BY THE DEGREE OF INFINITY TO WHICH THEY CONVERGE. THE DIAGRAM $\mathcal{E}$ MAY BE UNDERSTOOD AS A

"UNIVERSAL" (LAWVERE) FOR SUCH SITUATIONS.

IX: AS WILL SOON BECOME APPARENT, CLASSIFICATIONS OF ARROWS MAY BE CONSIDERED THE MAIN PROBLEM OF CONSTRUCTIVE CONCEPTUALISM IN CARTESIAN CLOSED CATEGORIES, WHERE C IS CONCEPT OR CATEGORY. THAT IS, WE WILL LOOK AT MORPHISMS OF THE (DEVELOPMENTAL) KIND $C \longleftarrow CC \longleftarrow CCC \longleftarrow$ .... (VIDE INTIMATE PEDAGOGY, EXERCISES IN THE ARTS OF BELLES LETTRES!).

GENERALLY, WHAT IS INTENDED IS THAT ARROWS GO PROXY FOR THE MEANING RELATION OBTAINING BETWEEN THE TERMS SITUATED ON BOTH SIDES OF THE ARROW-SIGN. THERE ARE, GENERALLY, NO RESTRICTIONS ON THE KIND OF OBJECTS THAT ARE PERMITTED TO APPEAR AS (THE) FIXED VALUES OF THE TERMS. THE COLLECTION OF ALL OBJECTS WHICH MAY BE SUBSTITUTED FOR THE VALUES OF THE TERMS OF THE ARROW-SIGNS, IS CALLED THE UNIVERSE, $\mathbf{U}$, WHICH THE MEANING RELATION WILL BE SAID TO REFER TO, WHENEVER INTERPRETED.

THE MEANS FOR DESIGNATING TERMS IN THEIR OCCURRENCES IN SOME COLLECTION OF ARROW-SIGNS REQUIRE ACCESS TO SOME VOCABULARY, $\mathbf{V}$, BY WHICH THESE DESIGNATIONS BECOME EXPRESSED. BY $\mathbf{L_V}$ WE DESIGNATE THE LANGUAGE GENERATED OVER $\mathbf{V}$, I.E. ALL FINITE SEQUENCES OF SYMBOLS OF $\mathbf{V}$. $\mathbf{G_L}$ SHALL DESIGNATE THE GRAMMAR GENERATING $\mathbf{L_V}$. (FOR FURTHER DETAILS, SEE THE THEORY OF SEMIOTICS).

X: WHEN WE ARE CONSIDERING THE ABSTRACT MECHANISMS OF OUR LANGUAGE OR OF COLLECTIONS OF SIGNS IN GENERAL, THEIR UNDERLYING REALITY OR "DEEP STRUCTURE" IS WHAT ESSENTIALLY CONTRIBUTES TO THE SENSE OR MEANING OF THESE MECHANISMS. FOR EACH FRAGMENT OF A LANGUAGE OR COLLECTION OF SIGNS, WE HAVE TO CONSIDER WHAT KIND OF WORLD OR FORM OF LIFE IS DEPICTED BY THIS FRAGMENT (ITS MODEL(S)). DEPENDING ON THE CONTEXT, THE MEANING

THAT A SIGN RECEIVES MAY BE CALLED THE WORLD-LOCATION OR LOCI OF THAT SIGN. IF THE SIGN IS A LOGICAL SIGN, WE MAY ALSO SPEAK OF THE LOGICAL COORDINATES AS THE MEANING(S) THAT THE SIGN RECEIVES.

THESE LOCATIONS OR COORDINATES MAKE UP THE CORE OF THE UNDERLYING REALITY OF OUR SIGNS. REALITY BEING A RESULT OF AUTHORITY, TO WIT, THE RESULT IMPOSED BY ITS AUTHOR(S), THE LIFE OF THIS "REALITY" IS RESTRICTED BY THE STANDARDS OF ETHICS WHICH THE AUTHOR LEGISLATES BY HIS DEMARCATION BETWEEN ADMISSIBLE AND INADMISSIBLE SYMBOLS AND SIGNS. THE MENTAL VACUUM OF OUR CONVENTIONALISTICALLY SUSTAINED CULTURE IS BUT A SECOND-ORDER CONSEQUENCE OF SOCIETY'S FALLACIOUSLY DEVELOPED MODALITIES. IN ORDER TO CORRECT OUR FIRST IMPRESSIONS OF THIS STATE OF AFFAIRS IT IS ADVISABLE TO LOOK AT SOME OF THE DEEPER MODALITIES OF OUR EXISTENCE, AS EXEMPLIFIED BY THE DEONTIC AND OPTATIVE ONES.

CONSEQUENTLY, WE MAY NOTE THAT CONNECTED WITH THE UNDERLYING REALITY OF THE USE OF SIGNS IS WHAT MAY BE TERMED RELEVANCE THEORY, ID EST THE STUDY OF AIMS AND MEANS ASSOCIATED WITH ANY REALITY. SOME SIGNS MAY LACK A MEANING SIMPLY BECAUSE THEY WERE INTENDED AS AN ACT OR GESTURE, WHICH, OF COURSE, MAY LACK MEANING (IN A WELL-DEFINED SENSE) BUT MUST BE ASSOCIATED WITH AN AIM. AN AIM IS ANY DESIRE TO ACCOMPLISH A CONSTRUCTION BY SOME GIVEN MEANS. IF THE LATTER ARE LACKING (OR INSUFFICIENT) WE SHALL SPEAK ABOUT IDEALS AND INSTEAD OF A CONSTRUCTION WE SHALL SPEAK OF A TENDENCY (TOWARDS THE IDEAL).

ONE OF THE BASIC TENETS OF OUR (ELEUTHERIC) RELEVANCE THEORY IS ITS COMMITMENT TO JUST AND PURPOSEFUL ACTS OF COMMUNICATION, ID EST ACTS FREED FROM OBSTACLES, FRAUD AND COERCION. OBVIOUSLY, THIS PRINCIPLE IS CLEARLY RELATED TO IDEAS IN ETHICS AS WELL AS EPISTEMOLOGY (OF CERTAINTY) AND IT CON-

STITUTES A CORNERSTONE OF OUR GENERAL THEORY OF MODALITIES, WHICH WILL BE DEVELOPED IN A SUBSEQUENT PAPER.

PURPOSEFULNESS MEANS, ABOVE ALL, THAT THE MEANS ASSOCIATED WITH THE AIMS ARE SUFFICIENT FOR ACHIEVING THE AIMS. THAT IS TO SAY, AN AIM WHICH IS NOT REMAINING AN IDEAL IS BY DEFINITION WITHOUT PURPOSE IF IT LACKS THE NECESSARY AND SUFFICIENT MEANS ASSOCIATED WITH ITS FULFILLABILITY. OF COURSE, IF AN AIM IS FOUND PURPOSELESS, IT CAN STILL BE RETAINED, IF DESIRED, AS AN IDEAL. BUT THEN THE IDEAL IS PROPERLY CALLED IDLE UNLESS IT BECOMES (OR ALREADY IS) ASSOCIATED WITH A SEARCH FOR MEANS THAT SUFFICE FOR SUSTAINING THE TENDENCY ON WHICH THE IDEAL DEPENDS.

THE JUSTNESS OF AN ACT IS SIMPLY ITS ADMISSIBILITY IN THE CONTEXT OF ITS APPEARANCE. FOR INSTANCE, IT IS REQUIRED THAT A NEW ACT IS COMPATIBLE WITH PREVIOUSLY COMMITTED PURPOSEFUL ACTS IN THE SENSE THAT IT DOES NOT OBSTRUCT ALREADY ACHIEVED ACTS OF COMMUNICATION. IF AN OBSTRUCTION WOULD FOLLOW FROM SOME SPECIFIC ACT IN $\mathcal{A}$, THEN THIS FACT WOULD BE AN INDICATION OF THE FAULTINESS OF THE LOGIC BEHIND THE ACTIVITY, WHICH, ACCORDINGLY, ONE IS OBLIGED TO REVISE OR SIMPLY ABANDON IN ORDER TO SECURE THE ELEUTHERIC CONTINUATION OF $\mathcal{A}$.

XI. ACTIVITIES DECOMPOSE INTO INDIVIDUAL SEQUENCES OF ACTS WHICH ULTIMATELY EXHAUST THE ACTIVITY. AMONG THE "ATOMIC" ACTS YIELDED UNDER ANY SUCH DECOMPOSITION IT IS DESIRABLE (IN ORDER TO AVOID CONFUSION) TO DISTINGUISH BETWEEN ORGANIC ACTS, I.E. THOSE CONNECTED WITH INTERFERENCE WITH THE PHYSICAL WORLD, AND EPISTEMIC ACTS, WHICH INVOLVE INTERFERENCE WITH OUR ABSTRACTION CAPABILITIES, I.E. THOSE ACTS WHICH MAY BE DESCRIBED AS PURELY MENTAL. AS IS PLAIN, CONSTRUCTIVE CONCEPTUALISM FOCUSES MAINLY ON ACTIVITIES WHERE THE LATTER ACTS DOMINATE. USUALLY,

AS OF THIS KIND ARE REFERRED TO AS THOUGHT PROCESSES AND CODED IN THEIR EXTENSIVENESS BY THE UNIVERSAL $\mathcal{E}$-DIAGRAM.

AN APPEARANCE OF A SIGN HAS AT LEAST TWO EPISTEMIC STRUCTURES ASSOCIATED WITH IT, VIZ. ITS SURFACE STRUCTURE AND ITS DEEP STRUCTURE, RESPECTIVELY. ONLY IN A LOGICALLY PERFECTED LANGUAGE (A SATURATED LANGUAGE) DO THESE TWO STRUCTURES COINCIDE. IN OTHER LANGUAGES THE COINCIDENCE MAY VARY, AND IT BELONGS TO THE TACTICS OF ATTENTION OF THOSE LANGUAGES TO DETERMINE THE EXACT PROJECTION OF SURFACE FORM ONTO ITS COMPANION DEEP STRUCTURE FORM.

THE PROJECTION RULES OF THE SIGN DETERMINE ITS WORLD LOCATION AND THUS GIVE US THE COORDINATES OF THE SIGN'S UNDERLYING REALITY. THIS IS PART OF THE CRITERION FOR THE MEANINGFULNESS OF THE SIGN'S APPEARANCE AND IT IS ALSO PART OF THE CRITERION FOR EVERY INTRODUCTORY OCCURRENCE OF A SIGN, IN WHICH THE RULES OF PROJECTIONS ARE LAID DOWN.

THE ACQUISITION AND USE OF A SIGN IS A HIGHLY INTRICATE NETWORK OF MENTAL PROCESSES CONDUCTED UNDER VARIOUS SPECTRA OF MODALITIES WHICH OPERATE ON ALL ACTIONS THAT REQUIRE PARTICIPATION UNDER THE RULES OF USE OF SIGNS. IT IS RATHER A TRAGEDY THAT PEOPLE HAVE OVERLOOKED THE DEEP CONTRIBUTION OF THESE MODALITIES TO THE FUNCTIONAL AND CREATIVE ASPECTS OF THE USE OF SIGNS, WHICH, INCIDENTALLY, PARTIALLY ACCOUNTS FOR THE DETERIORATING STANDARDS NOT ONLY OF COLLOQUIAL AND POLITICAL LANGUAGE BUT ALSO OF SUCH HIGHLY STYLIZED FORMS OF CONDUCT AS ARE PRESCRIBED BY THE CODES OF JURISPRUDENCE AND INTERNATIONAL LAW.

DIGRESSION: PRESENT STANDARDS ARE TRULY A POOR MONUMENT TO THE INTELLECTUAL ACHIEVEMENTS OF THIS CENTURY AND THERE IS EVEN AN URGENCY TO CALL FOR SOME (FUNDAMENTALISTIC) REGIMENTATION ON THE PART OF OUR SEMANTICS SO THAT THESE ACHIEVEMENTS MAY NOT

DETERIORATE INTO OBLIVION. A PLEA FOR A CONVIVIAL SEMANTICS ON A PAR WITH ILLICH'S PROGRAM FOR A CONVIVIAL RETOOLING OF OUR SOCIETY IS CERTAINLY CALLED FOR, IF THE LATTER AIM SHALL NOT REMAIN AN (IDLE) IDEAL. IN PARTICULAR, THE FACT THAT OUR LANGUAGE (IN DISTINCTION TO THEIRS) IS ALSO ONE OF OUR MOST PRECIOUS TOOLS (FOR ANY AIM) CANNOT BE EMPHASIZED SUFFICIENTLY IN THIS CONTEXT. (WARNING: DON'T CONFUSE THIS PLEA WITH ANY CONVENTIONALISTIC REVISION OF OUR SPEECH HABITS!)

XII. TO SUM UP. ONE OF OUR BASIC AIMS IS TO IMPLEMENT A FAMILY OF LANGUAGES $\mathfrak{L}$, WHICH MAY REFLECT BY ITS VERY DESIGN THE CONDITIONS FOR PURPOSEFUL ACTS OF COMMUNICATION. SUCH AN AIM OF IMPLEMENTING OUR SEMIOTICAL CAPABILITIES INVOLVES A CERTAIN AMOUNT OF "DEBUGGING" OF SOME (AND ULTIMATELY, ALL) OF THE IDLE SEMANTICAL OR HABITUAL CIRCUITS THAT PREVENT THE MIND FROM A CLEARED ACCESS TO ITS FUNDAMENTAL(ISTIC) CREATIVE POTENTIALS.

WITHIN SOME MODALITIES THE VERY DEBUGGING ACTIVITY ITSELF RESULTS IN A REFINEMENT OF OUR LANGUAGE TO THE DEGREE THAT ITS DESIRED IMPLEMENTATION HAS BECOME FULFILLED WHEN SUCCESSFULLY KILLING OFF THE BUGS.

OTHER MODALITIES, HOWEVER, REQUIRE THAT MORE CONSTRUCTIVE PROCESSES (MEDIATED BY INTIMATE PEDAGOGY) TAKE THE PLACE OF THOSE "FREE" PLACES ALONG THE DEBUGGED CIRCUITS. GENERALLY, THESE PROCESSES WILL NOT REMAIN FIXED AT THEIR CORRESPONDING SUBSTITUTED PLACES, BUT WILL PARTICIPATE IN THE INTEGRATION OF OTHER STREAMS OF CONSCIOUSNESS. (OBVIOUSLY, THE "FLOATING" CHARACTER OF THIS ACTIVITY DEMANDS ELEUTHERIA FOR THE SAKE OF ITS STABILITY AND (NON-PARALYZED) DEVELOPMENT). THIS PARTICIPATORY FUNCTION WILL BE REGARDED AS THE MAIN SOURCE FOR THE ARISING OF NEW MODALITIES AND WITH THE LATTER NEW MOODS OF

CONSCIOUSNESS AND THEIR ASSOCIATED TACTICS OF ATTENTION.

DIGRESSION: AS PEOPLE SEEM TO HAVE FAILED TO NOTICE, ONE OF THE MAIN OBSTACLES FOR THE REALIZATION OF THIS AIM IS THE OTHERWISE WELL-KNOWN CIRCUMSTANCE THAT IN ART (AND ELSEWHERE), WE ARE USUALLY INVOLVED WITH INTERPRETATIONS THAT YIELD TO MULTIPLE FORMS OF UNDERSTANDING, I.E. MOODS OF UNDERSTANDING THAT LACK UNIQUENESS (ONE OF THE CAUSES BEING, OF COURSE, THE "FREE FALL" INTO ASSOCIATIVE THINKING). CONSEQUENTLY, WHEN WE TRY TO FIX, IN SOME WAY, THIS UNDERSTANDING, WE FAIL TO PROVIDE FOR EVEN THE MOST ELEMENTARY ASPECTS OF $\mathrm{FREEDOM}_1$ AND $\mathrm{FREEDOM}_2$, THEREBY DEPRIVING US OF THE POSSIBILITY OF PROVIDING FOR JUST AND PURPOSEFUL ACTS OF COMMUNICATION IN A FIELD THAT PURPORTS TO DEFEND THESE VERY VALUES. THUS, NO FIXED MEANING CAN SERIOUSLY BE CLAIMED FOR WORKS OF ART OR ANY OTHER SYMBOLIC ACTIVITY (INCLUDING JURISPRUDENCE AND CODIFICATIONS OF HUMAN RIGHTS) THAT SOCIETY HAS PROVIDED FOR UP TILL NOW. OF COURSE, THERE IS SOMETIMES (BUT NOT ALWAYS) A CERTAIN ECONOMICAL GAIN INVOLVED WHEN A MULTIPLE SET OF MEANINGS CAN BE COMPRESSED INTO A SINGLE SIGN. BUT IF THE CLARITY OF THE SIGN SUFFERS TOO MUCH DURING THE COURSE OF THIS CONCEPTUAL COMPRESSION, ITS PURPOSEFULNESS BECOMES DOUBTFUL, AND HENCE, CEASES TO FULFILL ITS PRESUMED ROLE (AS FAR AS THAT ROLE INVOLVES ANY EXACT PURPOSE).

REMARK: WHEN AN AUTHOR SHOWS THE ADDRESSEE A SIGN, AN ACT OF CONFIDENCE TOWARDS THE AUTHOR TAKES PLACE WHEN THE ADDRESSEE INTERPRETS THE SIGN. THIS CONFIDENCE IS EXPRESSED BY TACTICS FOR IDENTIFICATIONS AND DISTINCTIONS THAT THE ADDRESSEE BRINGS FORTH DURING THE ACT OF INTERPRETATION. IN ADDITION THE TACTICS OF ATTENTION, WHICH THE ALLOWANCE OR ADMISSIBILITY OF THE SIGN IN PART DEFINES, FURTHER RESTRICTS THE POSSIBLE MOODS OF UNDERSTANDING OF THE SIGN. ON THE OTHER HAND, TACTICS OF NEGLECT ARE DETERMINED BY THE SPHERE OF CONFIDENCE THAT EMANATES

FROM THE AUTHOR WHEN ADDRESSING THE ADDRESSEE. THAT IS TO SAY, WHAT IS NOT EXPLICITLY EXPRESSED BUT PRESUPPOSED BY CONTEXT OR OTHER RELEVANT CLUES, CAN ONLY BE RETRACED BY FOLLOWING THE TACTICS OF NEGLECT AUTHORIZED BY THE SITUATION IN WHICH THE ACT OF COMMUNICATION TAKES PLACE. ANY NEGLECT NOT AUTHORIZED BY THIS TACTIC WILL BE CONSIDERED A VIOLATION OF THE CONDITION OF FAIRNESS FOR JUST AND PURPOSEFUL ACTS OF COMMUNICATION AND CONSEQUENTLY INADMISSIBLE FOR THE CONTEXT IN QUESTION. IT IS IMPORTANT TO NOTE, HOWEVER, THAT ANY NEGLECT BASED ON ALIEN CIRCUMSTANCES PLAYS NO RELEVANT ROLE WHEN DECLARING A NEGLECT INADMISSIBLE. THE ALIENNESS OF CIRCUMSTANCES IS PRESCRIBED BY RELEVANCE THEORY AND THE ETHICAL DOCTRINE ASSOCIATED WITH IT.

S E M I O T I C S

1

SEMIOTICS - I

THE IDEA OF AN ABSTRACT LANGUAGE ORIGINATED WITH THE DISCOVERY OF SOME SIMPLE ALGEBRAIC PROPERTIES PERTAINING TO THE COMBINATORIAL OR SYNTACTICAL RULES GOVERNING THE PRODUCTIONS OF STRINGS OF SYMBOLS. ONE OF THE MOST ELEMENTARY ALGEBRAIC STRUCTURES INVOLVING THESE PROPERTIES IS THE SEMI GROUP WITH 1, GENERALLY DESIGNATED $\mathfrak{a} = \langle \mathcal{A}, \circ \rangle$, WHERE THE FIRST COORDINATE DESIGNATES THE DOMAIN OF THE SEMI GROUP (ALSO CALLED THE CARRIER OF THE SEMI GROUP) AND THE SECOND COORDINATE DESIGNATES AN ASSOCIATIVE OPERATION (CONCATENATION) CLOSED WITH RESPECT TO THE DOMAIN OF THE SEMI GROUP.

OPERATIONS PERFORMED ON A CARRIER $\mathcal{A}$ ARE TERMED PRODUCTIONS OF THE STRUCTURE $\mathfrak{a}$ AND THEY CORRESPOND TO SENTENCE FORMS AND OTHER GRAMMATICALLY SIGNIFICANT UNITS ON THE SYNTACTIC LEVEL.

CHAINS OF PRODUCTIONS OVER $\mathcal{A}$ ARE GIVEN STRUCTURAL DESCRIPTIONS IN TERMS OF TREES, $\mathcal{T}_{\mathcal{A}}$, WHERE THE "ROOT" DESIGNATES THE INITIAL PRODUCTION (OR "START SYMBOL") AND THE BRANCHINGS DESIGNATE THE SUCCESSIVE APPLICATIONS OF OPERATIONS ON PREVIOUSLY OBTAINED "WORDS". BY THE CLOSURE OF $\mathcal{A}$ WE MEAN ALL TREES $\mathcal{T}_{\mathcal{A}}$ SUCH THAT $\mathcal{T}_{\mathcal{A}}$ OBTAINS IN $\mathfrak{a}$. THIS CLOSURE CORRESPONDS TO THE LANGUAGE GENERATED BY $\mathfrak{a}$, DESIGNATED $\mathcal{L}(\mathcal{A})$.

BY GENERALIZING THE CONCEPT OF A SEMI GROUP WITH 1 WE MAY OBTAIN A GLOBAL PRESENTATION OF $\mathcal{L}(\mathcal{A})$. THIS GENERALIZATION BRINGS US TO THE CONCEPTS OF CATEGORY THEORY, ONE OF THE HIGHLIGHTS OF EXACT THINKING AFTER THE CREATION OF CANTOR'S "PARADISE". BY A CATEGORY $\mathbf{C}$ WE SHALL UNDERSTAND A COLLECTION OF OBJECTS, CORRESPONDING TO THE WORDS ON $\mathcal{A}$ ABOVE, TOGETHER WITH A COLLECTION OF MORPHISMS, CORRESPONDING TO THE PRODUCTIONS GENERATING $\mathcal{L}(\mathcal{A})$.

OBJECTS WILL BE DENOTED $\mathbf{A}$, $\mathbf{B}$, $\mathbf{\Gamma}$, Д, ... AND MORPHISMS Я, Ю, Э, ... FOR EACH PAIR OF OBJECTS $\mathbf{A}$, $\mathbf{B}$, THERE IS A SET $\mathbf{C}(\mathbf{A}, \mathbf{B})$ OF MORPHISMS Я CARRYING $\mathbf{A}$ TO $\mathbf{B}$, AND, IN ADDITION, FOR EACH $\mathbf{A}$ IN $\mathbf{C}$, AN <u>IDENTITY MORPHISM</u> $\epsilon_{\mathbf{A}}(\mathbf{A}, \mathbf{A})$, ALSO WRITTEN $\mathcal{J}_{\mathbf{A}}$.

NOW, IF THERE ARE THREE MORPHISMS Я, Ю, Э SUCH THAT

$\text{Я}: \mathbf{A} \Rightarrow \mathbf{B}$, $\text{Ю}: \mathbf{B} \Rightarrow \mathbf{\Gamma}$, $\text{Э}: \mathbf{\Gamma} \Rightarrow \text{Д}$

THEN THE <u>COMPOSITION</u> OF THEM SATISFIES

$$(\text{ЯЮ})\,\text{Э} = \text{Я}\,(\text{ЮЭ})$$

ALSO, IF $\text{Я}: \mathbf{A} \Rightarrow \mathbf{B}$, THEN $\mathcal{J}_{\mathbf{A}}\text{Я} = \text{Я} = \text{Я}\mathcal{J}_{\mathbf{B}}$

TO COMPLETE OUR DEFINITION WE REMARK THAT A CATEGORY $\mathbf{C}$ IS ALWAYS <u>CLOSED</u> UNDER ARBITRARY <u>COMPOSITIONS</u> OF MORPHISMS, I.E. IF

$\text{Я} \in \mathbf{C}(\mathbf{A}, \mathbf{B})$ AND $\text{Ю} \in \mathbf{C}(\mathbf{B}, \mathbf{\Gamma})$, THEN ALWAYS

$\text{ЯЮ} \in \mathbf{C}(\mathbf{A}, \mathbf{\Gamma})$

THE <u>SYNTAX</u> OF $\mathcal{L}(\mathcal{A})$ CAN NOW BE SPECIFIED AS A CATEGORY $\mathcal{F}$ WHERE THE OBJECTS ARE STRINGS OF LETTERS DRAWN FROM SOME FIXED ALPHABET AND THE MORPHISMS ARE DERIVATIONS (TREES) OF ONE STRING FROM ANOTHER.

BY A DERIVATION $\mathcal{D}$ WE SHALL NOW UNDERSTAND THE FOLLOWING ORDERED TRIPLE

$$\mathcal{D} = \Big\langle (\mathbf{A}_0, \ldots\ \mathbf{A}_n),\ (\text{Я}_0, \ldots\ \text{Я}_{n-1}),\ (\lambda_0 - \zeta_0, \ldots, \lambda_{n-1} - \zeta_{n-1}) \Big\rangle$$

WHERE

$(\mathbf{A}_0 \ldots\ldots, \mathbf{A}_n)$ DESIGNATES THE WORD COORDINATE,

$(\mathfrak{R}_0 \ldots\ldots, \mathfrak{R}_{n-1})$ IS THE DERIVATION COORDINATE, AND

$(\lambda_0 - \zeta_0, \ldots\ldots, \lambda_{n-1}\ \zeta_{n-1})$ IS THE NEIGHBOURHOOD COORDINATE,

THE LATTER GIVEN A TOPOLOGICAL INTERPRETATION (SEE BELOW).

THE LENGTH ZERO DERIVATION $\langle(\mathbf{A}),(\ \ ),(\ \ )\rangle$ IS REGARDED AS THE $\mathbf{A}$-IDENTITY DERIVATION, WHILE THE LENGTH ONE DERIVATION $\langle(\mathbf{A}, \mathbf{B}),(\mathfrak{R} : \mathbf{A} \Rightarrow \mathbf{B}), (\lambda - \zeta)\rangle$ WILL BE "ABBREVIATED" $\mathfrak{R} : \mathbf{A} \Rightarrow \mathbf{B}$ AND PURPOSELY CONFUSED WITH THE "REAL" PRODUCTION $\mathbf{A} \Rightarrow \mathbf{B}$ IN $\mathcal{L}(\mathcal{A})$. CLEARLY $\mathfrak{D}$ MAY BE TURNED INTO AN INDEPENDENT CATEGORY, ID EST INDEPENDENT OF $\mathcal{L}(\mathcal{A})$.

THE IMPORTANCE OF THE ABOVE CONCEPTS COMES FROM THE FACT THAT MORPHISMS ARE EXAMPLES OF A GENERAL CLASS OF ARROWS. ANOTHER EXAMPLE IS THE CLASS OF FUNCTORS THAT EXISTS BETWEEN CATEGORIES THEMSELVES.

A FUNCTOR Υ FROM A CATEGORY $\mathbf{C}_0$ TO A CATEGORY $\mathbf{C}_1$ IS SIMPLY TWO CLASSES OF ARROWS, ONE SENDING OBJECTS IN $\mathbf{C}_0$ TO OBJECTS IN $\mathbf{C}_1$ AND THE OTHER SENDING MORPHISMS IN $\mathbf{C}_0$ TO MORPHISMS IN $\mathbf{C}_1$. FORMALLY, THE FOLLOWING SITUATION OBTAINS: IF Υ IS A FUNCTOR BETWEEN $\mathbf{C}_0$ AND $\mathbf{C}_1$, ID EST

$$\Upsilon : \mathbf{C}_0 \longrightarrow \mathbf{C}_1$$

, THEN FOR EVERY $\mathbf{A} \in \mathbf{C}_0$, Υ ASSIGNS AN OBJECT $\Upsilon(\mathbf{A})$ IN $\mathbf{C}_1$ AND FOR EACH $\mathfrak{R} \in \mathbf{C}_0$ Υ ASSIGNS THE MORPHISM $\Upsilon(\mathfrak{R})$ IN $\mathbf{C}_1$.

Now, by the SEMANTICS of $\mathcal{L}(\mathfrak{A})$ we mean a CO-FUNCTOR

$$\mathcal{J} : \mathfrak{F} \Longrightarrow \mathfrak{U},$$

where $\mathfrak{U}$, of course, denotes the SEMANTIC CATEGORY associated with $\mathcal{L}(\mathfrak{A})$ when generated by the Syntax Category $\mathfrak{F}$.

More specifically, the Co-Functor $\mathcal{J}$ specifies an INTERPRETATION of $\mathfrak{F}$ by taking Objects to CARTESIAN PRODUCTS in $\mathfrak{U}$ and derivations to functions in $\mathfrak{U}$. In other words, the Semantic Category $\mathfrak{U}$ consists of a Category of SETS and FUNCTIONS and the image of the interpretation $\mathcal{J}$ is called the SEMANTICS of the interpretation. For example, the interpretation of an Object $\mathbf{A}_{\mathcal{F}}$ of the Syntax Category consists of those Functions $\mathbf{Ш}$ that are Contravariant in $\mathfrak{U}$. (They correspond to retracts in a Topos).

If $\mathbf{Я}$ is a Morphism in $\mathfrak{U}$ (i.e. $\mathbf{Я}$ is a Function), then the neighbourhood of $\mathbf{Я}$ is defined as those Morphisms $\mathbf{Я}_1, \ldots\ldots, \mathbf{Я}_n$ that act as Identities on the extended neighbourhood domain of $\mathbf{Я}$. That is, $\mathbf{Я}_1, \ldots\ldots, \mathbf{Я}_n$ are neighbourhoods of $\mathbf{Я}$ if the following diagrams commute:

$$\begin{tikzcd}
\mathbf{А} \times \mathbf{В} \times \mathbf{Г} \arrow[r] \arrow[d] \arrow[dr, phantom, "\mathbf{Я}_1, \ldots, \mathbf{Я}_n"] & \mathbf{А} \times \mathbf{Д} \times \mathbf{Г} \arrow[d] \\
\mathbf{В} \arrow[r] & \mathbf{Д}
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{А} \times \mathbf{В} \times \mathbf{Г} \arrow[rr] \arrow[dr] \arrow[rr, phantom, shift right=4ex, "Я_1 \ldots Я_n"] & & \mathbf{А} \times \mathbf{Д} \times \mathbf{Г} \arrow[dl] \\
& \mathbf{А} &
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{А} \times \mathbf{В} \times \mathbf{Г} \arrow[rr] \arrow[dr] & & \mathbf{А} \times \mathbf{Д} \times \mathbf{Г} \arrow[dl] \\
& \mathbf{Г} &
\end{tikzcd}$$

WHERE $\mathbf{Я}$ IS THE MORPHISM $\mathbf{Я} : \mathbf{В} \Rightarrow \mathbf{Д}$ AND X INDICATES CARTESIAN PRODUCT.

FURTHER EXAMPLES OF GENERAL CLASSES OF ARROWS TO BE MET WITH IN CATEGORY THEORY - BESIDES MORPHISMS AND FUNCTORS - ARE <u>MONO-MORPHISMS</u> AND <u>EPI-MORPHISMS</u>, AS IN THE FOLLOWING

<u>DEFINITION</u>. $\mathbf{Щ}$ IS A <u>MONO-MORPHISM</u> IN A CATEGORY $\mathbf{C}$, IF FOR ALL $\mathbf{Ю}, \mathbf{Э} \in \mathbf{C}$

$$\mathbf{Ю} : \mathbf{Г} \Rightarrow \mathbf{А} , \mathbf{Э} : \mathbf{Г} \Rightarrow \mathbf{А}$$

IMPLIES $\mathbf{Ю} = \mathbf{Э}$ IF $\mathbf{ЮЩ} = \mathbf{ЭЩ}$

DEFINITION. **Ш** IS AN EPI-MORPHISM IN A CATEGORY **C**, IF FOR ALL **Ю**, **Э** $\in$ **C**

$$\textbf{Ю} : \textbf{B} \Rightarrow \textbf{Г},\ \textbf{Э} : \textbf{B} \Rightarrow \textbf{Г}$$

IMPLIES $\quad \textbf{Ю} = \textbf{Э} \quad$ IF $\textbf{ШЮ} = \textbf{ШЭ}$

Щ AND **Ш** WILL BE OF IMPORTANCE FOR THE MAPPINGS OF THE INNER STRUCTURES OF TOPOSES.

TOPOSES GIVE RISE TO YET OTHER GENERAL CLASSES OF ARROWS LIKE PULL-BACKS AND PUSH-OUTS. FOR THE MOMENT WE WILL STOP THE PRESENT CLASSIFICATION OF ARROWS, ONLY MENTIONING ONE FURTHER CLASS, AND LEAVING THE OTHERS FOR ANOTHER OCCASION. THIS CLASS IS CALLED THE CLUB OPERATORS, Ω, WHICH ASSIGNS TO EACH OBJECT **A** A MORPHISM

$$\textbf{Ч}_{\mathbf{A}} : \Upsilon\mathbf{A} \longrightarrow \Phi\mathbf{A}$$

WHERE Υ AND Φ ARE **2**-FUNCTORS.

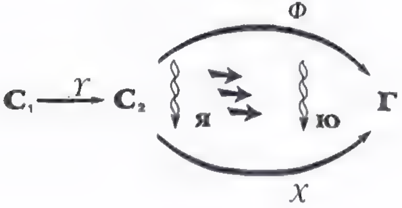
FURTHER, TO EACH MORPHISM **Ч** OF THE UNDERLYING **2**-CATEGORY **Г** OF **Д**, A **2**-CELL MORPHISM $\textbf{Ч}_{\mathbf{A}}$ IN THE CO-DOMAIN **2**-CATEGORY **Д** IS ASSIGNED SUCH THAT THE FOLLOWING "SQUARE" OBTAINS:

$$\begin{tikzcd}
\Upsilon\mathbf{A} \arrow[r] \arrow[d, "\Omega;"'] \arrow[dr, phantom, "\Downarrow \textbf{Ч}_{\mathbf{A}}"] & \Phi\mathbf{A} \arrow[d, "\Phi\textbf{Ч}"] \\
\Upsilon \Leftarrow \textbf{Ч}_{\mathbf{A}} \arrow[r] & \Phi \Leftarrow \textbf{Ч}_{\mathbf{A}}
\end{tikzcd}$$

INTUITIVELY, Ω IS A <u>NATURAL TRANSFORMATION</u> BETWEEN CATEGORIES. SPECIFIC NATURAL TRANSFORMATIONS WILL BE PICTURED:

$$\text{Ш}_{\Omega} : \Upsilon \rightsquigarrow \chi$$

THE CLUB OPERATOR PLAYS A PROMINENT ROLE FOR THE <u>PASSAGE</u> BETWEEN CATEGORIES, AS WHEN WE WISH TO GO FROM ONE SEMANTIC CATEGORY $\mathcal{U}_1$ TO ANOTHER SEMANTIC CATEGORY $\mathcal{U}_2$, WITH COMMON UNDERLYING SYNTACTIC CATEGORY $\mathcal{F}_0$. THE DEONTIC AXIOMS DETERMINING THE ADMISSIBLE PASSAGES FOR SOME Ω ARE CALLED THE <u>DOCTRINE OF THE CLUB OF CATEGORIES</u> (OR JUST <u>DOCTRINES FOR SCHOOLS</u>), WHERE THE "CLUB" NOTION NOW REFERS TO THE CATEGORIES IN THE DOMAIN AND CODOMAIN OF Ω. FOR EXAMPLE, A SITUATION THAT IS TYPICAL FOR ANY DOCTRINE FOR SCHOOLS IS THE OPERATION OF <u>PASTING</u> OBJECTS (HERE, CATEGORIES) IN A CLUB. SO ONE THING PERMITTED IS TO PASS FROM THE SITUATION



to the situation

$$\begin{tikzcd}[column sep=huge]
\mathbf{C}_1 \arrow[r, bend left=50, "\Upsilon\Phi"] \arrow[r, bend right=50, "\Upsilon X"'] \arrow[r, phantom, "\Downarrow Я_{\Upsilon} \;\Rrightarrow\; \Downarrow Ю_{\Upsilon}"] & \mathbf{\Gamma}
\end{tikzcd}$$

where $\Rrightarrow$ denotes the pasting operation and $\mathbf{\Gamma}$ is an element of the Club.

Diagrams of Concept Formation Processes underlying Forests of Correct Reasonings make heavy use of $\Rrightarrow$, especially when the club has many-sorted Doctrines for Schools.

\+ \+ \+ \+ \+

<u>Some elementary categories</u> (Arrow and Time Categories).

(1) Let $\mathcal{V}$ be a (small) universe of <u>sets</u>. The <u>arrow category</u>

$$\mathcal{V}^{\Rightarrow}$$

will consist of those Objects $\lambda : \mathbf{X}_0 \Rightarrow \mathbf{X}_1$ for which $\mathbf{X}_0$ is a domain for λ and $\mathbf{X}_1$ its

co-domain and where the Morphisms, i.e. Arrows

$$\Lambda : \lambda : \mathbf{X}_0 \Rightarrow \mathbf{X}_1 \Rightarrow \lambda' : \mathbf{Y}_0 \Rightarrow \mathbf{Y}_1 \text{, are}$$

the pairs of functions λ_0, λ_1 such that the following diagram commutes, i.e. $\lambda\ \Lambda^0 = \Lambda'\ \lambda$

$$\begin{tikzcd}
\mathbf{X}_0 \arrow[r, "\lambda_0"] \arrow[d, "\Lambda^0"'] & \mathbf{X}_1 \arrow[d, "\Lambda'"] \\
\mathbf{Y}_0 \arrow[r, "\lambda_1"] & \mathbf{Y}_1
\end{tikzcd}$$

(2) If $\mathcal{P}$ is a (discrete) <u>process</u>, then $\mathcal{V}^{\mathcal{P}}$ is the Category of Times (relative $\mathcal{P}$). If instead of $\mathcal{V}$ and/or $\mathcal{P}$ we put the Category on the forms

$$\mathcal{V}^{\mathcal{E}} \quad \text{or} \quad \mathcal{E}^{\mathcal{E}},$$

we get the (universal) Categories of local and global Time, respectively.

For the Category $\mathcal{V}^{\mathcal{P}}$ we shall have as Objects infinitely proceeding sequences or strings

$$\mathbf{X}_0 \Rightarrow \mathbf{X}_1 \Rightarrow \mathbf{X}_2 \Rightarrow \cdots \Rightarrow \mathbf{X}_m \Rightarrow \cdots$$

of Arrows between sets $\mathbf{X}_j \in \mathcal{V}$ and as Morphisms sequences of Λ-Arrows, like the following:

$$
\begin{tikzcd}
\mathbf{X}_0 \arrow[r, "\lambda"] \arrow[d, "\Lambda_0"'] & \mathbf{X}_1 \arrow[r, "\lambda"] \arrow[d, "\Lambda_1"'] & \mathbf{X}_2 \arrow[r, "\lambda"] \arrow[d, "\Lambda_2"'] & \mathbf{X}_3 \arrow[r, "\lambda"] \arrow[d, "\Lambda_3"'] & \cdots \\
\mathbf{Y}_0 \arrow[r, "\lambda"] & \mathbf{Y}_1 \arrow[r, "\lambda"] & \mathbf{Y}_2 \arrow[r, "\lambda"] & \mathbf{Y}_3 \arrow[r, "\lambda"] & \cdots
\end{tikzcd}
$$

In terms of Arrows, $\mathcal{V}^{\Rightarrow}$ and $\mathcal{V}^{\mathcal{P}}$ differ in the following way

$$\mathcal{V}^{\Rightarrow} \; : \bullet \Rightarrow \bullet$$

$$\mathcal{V}^{\mathcal{P}} \; : \bullet \Rightarrow \bullet \Rightarrow \bullet \Rightarrow \; \bullet \bullet \bullet$$

<u>The definitive category of all categories</u>.

For Toposes as well as elsewhere, the DOCTRINAL CATEGORY $\mathcal{E}$ is the <u>FORGETFUL CO-DOMAIN CATEGORY</u> of the <u>UNIVERSAL FORGETFUL FUNCTOR</u>

$$\overset{\#}{\mathcal{E}}$$

Its Objects are many-sorted - Brackets, $[\;\;]$, and Cups, $\sqcup$ - and the Morphisms are obtained through iterations in the Cumulative Hierarchy $\mathcal{V}$ of Arrow Operations $\longrightarrow$ for denotational connections between Objects in $\mathcal{E}$.

Any diagram for a Concept Formation Process (guided by Correct Reasonings) has an <u>ADJOINTED FORGETFUL CO-FUNCTOR</u>

mapping elements in the Process to Cartesian Products of Objects in $\mathcal{E}$ and Morphisms to Functions (Arrows) in $\mathcal{E}$. Evidently, Cartesian Closed Categories (CCC's) will be the most prominent tool for constructing Toposes. By this last step we attempt a move from Lawvere's <u>OBJECTIVE DIALECTICS</u> to <u>NATURAL DIALECTICS</u> where Club Operations and other Natural Transformations ($\rightsquigarrow$) dominate over the specifications of Category Objects. Clearly, $\mathcal{E}$ considered as a Category will be our alternative to the Category of all Categories as a foundation of mathematics and Generalized Constructive Conceptualism.

Of course, $\mathcal{E}$ and its Sub-Objects will still serve as our favorite example of Inaccessible Cardinals, Toposes, Cosmois and what not that will be encountered in the pursuit of the delights of Exact Thinking.

S E M I O T I C S 2

SEMIOTICS - II

In order to understand our idea of a Topos it is necessary to introduce some notions from TOPOLOGY, i.e. the theory of INVISIBLE SPACES. In particular, Toposes are intended as exact formalizations of TACTICS OF PARTIALLY ORDERED SPACES, abbreviated as TOPOS.

Corresponding to the CONTINUOUS vs. DISCRETE Modes of Thinking we introduce CONTINUOUS and DISCRETE SPACES. Continuous Spaces are termed MEASURABLE CANTOR SPACES generated on the core of a Continuous Product Topology (C-Spaces). Discrete Spaces are of a Boolean kind and are represented by, for example, the Measurable 2-Space. Clearly, from a semiotical point of view, the interpretation differentiating between a Space Sign $\square$ of the kind C- or 2- , respectively, is determined by the TACTIC OF ATTENTION on the ORDER of $\square$.

Beyond the above (Black and White) dichotomy of Spaces and their Topologies, we wish to recognize some additional "chromatic" characteristics. Thus we shall distinguish between point - Spaces with properties such as COMPACTNESS, CONNECTEDNESS, SEPARABILITY and their negative counterparts INCOMPACTNESS, UNCONNECTEDNESS and INSEPARABILITY, respectively. It turns out that all spaces $\mathbf{T}_\square$ may effectively be determined by such properties alone and that the combinations of them determine the CHROMATICS induced by degrees of attention directed towards Spaces $\mathbf{T}_\square$ of exact Thinking. We prefer to call such Spaces $\mathbf{T}_\square$ CHROMATIC TOPOLOGIES when, and only when, the fixing of Arrows for their points and basis are given in terms of the above properties.

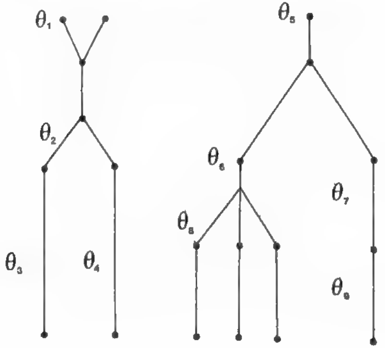
DIGRESSION: CHROMATIC THREADS

SPACES WITH POINT-BASES ARE EXAMPLES OF CHAINS OF CORRECT REASONINGS WHERE THE CHAINS COME OUT AS CHROMATIC THREADS.

LET $\theta_1, \ldots\ldots, \theta_\omega$ BE A CHAIN OF CORRECT REASONING. EACH θ_i, $i \in \omega$, IS A COLLECTION OF $\vdash$ -SENTENCES (IN THE STYLE OF FREGE), EACH ONE BEING OF THE FORM $\Gamma \vdash \Delta$, WHERE Γ, Δ ARE COLLECTIONS OF SENTENCES (I.E. OBJECTS IN A STRUCTURE $\mathfrak{a}$). $\Gamma \vdash \Delta$ IS READ $\mathfrak{R}_\Delta \Gamma$, I.E. THERE IS A (NOT NECESSARILY UNIQUE) CORRECT REASONING $\mathfrak{R}$ FOR OBTAINING Δ FROM Γ.

THE CORRECTNESS OF $\mathfrak{R}$ DEPENDS MAINLY ON THE POSSIBILITY OF ASSIGNING ARGUMENTAL SUPPORTS TO THE OCCURRENCES OF $\vdash$ -SYMBOLS BELONGING TO θ_i, FOR ALL $i \in \omega$. THE A.S. OF A $\vdash$ -SENTENCE IS A JUSTIFICATION FOR THE ACCEPTANCE OF THE REASONING $\mathfrak{R}$ REFERRED TO BY THE OCCURRENCE OF $\vdash$ IN THE CONTEXT OF A $\vdash$ - SENTENCE. THE TOTALITY OF A.S.'S ASSIGNED TO A CHAIN OF CORRECT REASONING θ IS CALLED THE ENVELOPE OF θ. THE ENVELOPE IS CHROMATIC IF ITS THREADS ARE.

θ:



(End of Digression)

Besides the Topological notions considered above, Semiotics may be considered as providing for those <u>construction procedures</u> $\mathcal{M}$ required to obtain a fixed <u>text</u> $\mathcal{T}$.

By a construction procedure $\mathcal{M}$ for $\mathcal{T}$ we shall understand a collection of <u>rules</u>, $\mathcal{R}$, and <u>methods</u>, $\mathscr{M}$, such that any part $\mathcal{T}'$ of $\mathcal{T}$ is obtained by some following of $\mathcal{R}$, $\mathscr{M}$. Intuitively, $\mathcal{R}$ is the <u>logic</u> for the means $\mathscr{M}$ when aimed at $\mathcal{T}$, i.e. the Envelope for $\mathcal{M}$.

If a language $\mathcal{L}$ is understood as a method for introducing and eliminating signs, each appearance of a sign σ will be connected with a method $\mathcal{L}\{\sigma\}$ in effect of which the appearance of σ is obtained.

<u>The following of a language</u> $\mathcal{L}$ is indicated by certain constructions of parts of the objects we have designated $\mathcal{T}$. Generally, an object $\mathcal{T}$ may be obtained through the following of a set $\mathcal{L}_1, \ldots\ldots, \mathcal{L}_m$ of languages. For each $\mathcal{L}_i$, $1 \leq i \leq m$, there shall correspond a unique set of parts $\mathcal{T}_{i_1}, \ldots\ldots, \mathcal{T}_{i_k} \in \mathcal{T}_i$ such that for each $\mathcal{T}_{i_j}$, if $\mathcal{L}_i\{\mathcal{T}_{i_j}\}$ <u>constructs</u> the part $\mathcal{T}_{i_j}'$, then $\mathcal{L}_i\{\mathcal{T}_i\}$ <u>constructs</u> $\mathcal{T}$ (the partial image of $\mathcal{L}_i$).

$\mathcal{L}$ may be considered as the collection of all $\mathcal{L}\{\sigma\}$ for which $\mathcal{L}$ constructs σ.

If $\mathcal{L}\{\sigma\}$ and a following of $\mathcal{L}\{\sigma\}$ is indicated, $\Diamond\left[\mathcal{T}_{i_j}(\sigma)\right]$, then $\mathcal{T}_{i_j}$ is said to be <u>clear in the sign</u> σ. The same applies to $\mathcal{T}_{i_j}(\bar{\sigma})$, $\mathcal{T}_i(\bar{\sigma})$ and $\mathcal{T}(\bar{\sigma})$. Consequently, if $\bar{\sigma} = \sigma_1, \ldots\ldots, \sigma_n$ exhausts the signs

APPEARING THROUGH $\mathcal{T}_1$ (σ_1 , ... σ_n) AND $\mathcal{T}$ IS CLEAR IN EACH σ_i FOR ALL APPEARANCES THROUGH $\mathcal{T}_i$, THEN $\mathcal{T}$ IS SAID TO BE CLEAR IN ALL OF ITS SIGNS.

BY A <u>SEMIOTICS</u> OF A (PRE-)THEORY FOR A TEXT $\mathcal{T}$, TH($\mathcal{T}$), WE SHALL UNDERSTAND A METHOD $\mathfrak{m}$ OF A CLASS OF CONSTRUCTION PROCEDURES $\mathcal{M}$ SUCH THAT

(1) $\mathrm{R}_{\mathfrak{s}} \subset \mathcal{M}$

(2) ANY APPLICATION OF AN ELEMENT $\mathfrak{r} \in \mathrm{R}_{\mathfrak{s}}$ ON ANY PREVIOUSLY CONSTRUCTED OBJECTS $\mathcal{T}_1$, ... $\mathcal{T}_k$ IS <u>SECURED</u> (I.E. PERMITTED) IF THE RESULT OF APPLYING $\mathfrak{r}$ TO $\mathcal{T}_1$, ... $\mathcal{T}_k$ ONLY DEPENDS ON $\mathfrak{m}_1$, ... $\mathfrak{m}_1 \in \mathcal{M}$ AND PARTS $\mathcal{T}_1'$, ... $\mathcal{T}_k'$ SUCH THAT $\mathfrak{m}_1(\mathcal{T}_1') = \mathcal{T}_1$, ... $\mathfrak{m}_1(\mathcal{T}_k') = \mathcal{T}_k$

(3) IF $\mathcal{T}'$ IS ANY PART OF $\mathcal{T}$, THEN THERE ARE $\mathfrak{m}_1$, ... $\mathfrak{m}_k \in \mathcal{M}$ SUCH THAT FOR ALL PARTS $\mathcal{T}''$ PRECEDING $\mathcal{T}'$, $\mathfrak{m}_1$, ... $\mathfrak{m}_k$ ($\mathcal{T}''$) = $\mathcal{T}'$.

LET $\mathcal{T}$ BE A FIXED TEXT. IF $\mathcal{T}$ IS <u>CLEAR</u>, THEN $\mathcal{T}$ MIGHT BE UNDERSTOOD AS A <u>DISCRETE PROCESS</u> $\Pi\mathcal{T}$. IN PARTICULAR, THE FOLLOWING OF $\mathcal{L}$ ($\mathcal{T}$) FOR CLEAR $\mathcal{T}$ MIGHT BE UNDERSTOOD AS THE FOLLOWING OF A <u>CLEAR LANGUAGE</u>.

AS EXPECTED, THERE IS NO DECISION PROCEDURE FOR RECOGNIZING CLEAR $\mathcal{T}$'S. SO TO SHOW THAT $\mathcal{T}$ IS CLEAR, IT IS NECESSARY TO CONSTRUCT A METHOD $\mathcal{L}$ SUCH THAT $\mathcal{L}$ ($\mathcal{T}$) <u>DISCRETELY</u> CONSTRUCTS $\mathcal{T}$. GIVEN $\mathcal{T}$, THE <u>MEANING PROBLEM</u> FOR $\mathcal{T}$ IS THE PROBLEM OF RECOVERING A SUITABLE $\mathcal{L}$ SUCH THAT $\mathcal{L}$ ($\mathcal{T}$) CONSTRUCTS $\mathcal{T}$ DISCRETELY.

THE FOLLOWING SCHEME IS FOLLOWED IN EVERY SITUATION ESTABLISH

ing denotational connections between sign-events of discrete character.

$$\begin{bmatrix} \alpha_1 \longrightarrow \alpha_2 \\ \alpha_2 \longrightarrow \alpha_3 \\ \alpha_3 \longrightarrow \alpha_4 \\ \vdots \\ \alpha_{n-1} \longrightarrow \alpha_n \end{bmatrix} \rightarrowtail \alpha_1 \longrightarrow \alpha_n$$

The above schema generates the admissible fibrations stemming from iterative denotational indications. At its origin the schema indicates the connection between the sign α_1 and all values α_2 of α_1. The values α_2 are the <u>points</u> or <u>meanings</u> of α_1. In order for α_1 to be <u>definite</u>, the connection $\alpha_i \longrightarrow \alpha_j$ must be clear, id est the pointing of α_1 to α_j must be constructed as $\mathcal{M}(\alpha_i \longrightarrow \alpha_j)$ and, in addition, α_j must be a singleton or a Terminal Object. Every sign for Toposes is definite by definition of an Exact Language and, consequently, collapses on the initial Noeama π° when the correct logical analysis is provided for.

<u>Warning</u>: Our use of the notion Th($\mathcal{T}$) or of the term "Semiotics" is totally independent of the present French fashion of the "literary" study of "theories" of signs (vide Barth et al.)

TOPOSES

SHEAVES

&

ADJOINTS

TOPOSES.

SHEAVES AND ADJOINTS

AN INITIAL POINT OF THE SUPPORT OF AN ABSTRACT CONCEPT FORMATION PROCESS Π IS CALLED THE NOEMA OF Π AND IS DENOTED Π^{o} (IN TERMS OF CCC'S, Π^{o} IS A TERMINAL OBJECT OF THE ASSOCIATED CATEGORY).

THE CHAIN(S) OF ARROWS ORIGINATING AT A NOEMA Π^{o} AND ASCENDING IN THE DIAGRAM OF Σ, I.E. $\Delta(\Sigma)$, IS THE FIBRATION Φ ASSOCIATED WITH Π^{o}.

A FIBER BUNDLE Φ IS A NETWORK OR LATTICE OF FIBRATIONS Φ_1, ..., Φ_n, ... ORIGINATING ON DIFFERENT NOEMAS AND ACTING ON THE FORMATION OF A PRE-TOPOS, WHERE A PRE-TOPOS IS A NON-EMPTY COLLECTION OF SITES IN A TOPOS (THE EXTERNAL CATEGORY RELATIVE THE PRE-TOPOS FORMATION). GENERALLY, SITES ARE IDENTIFIED BY THE IMAGE OF THE CONTINUOUS FIXED-POINT FUNCTIONS ON THE FIBER BUNDLES UNDERLYING A CORRECT REASONING IN CCC CONTEXTS.

FOR EACH PAIR OF OBJECTS $\left|\mathbf{a}_i\right|^k$, $\left|\mathbf{a}_j\right|^m$ PARTICIPATING IN A FIBRATION Φ, THE SUB-OBJECT FIBER OR ARROW CONNECTING THEM IS CALLED A PRE-MORPHISM. A COMPLETE MORPHISM IS A FAMILY OF PRE-MORPHISMS BELONGING TO A FIBER BUNDLE Φ OF AN ABSTRACT CONCEPT FORMATION PROCESS Π.

A TOPOS, FINALLY, CONSISTS OF A COLLECTION OF OBJECTS, THE SITES, TOGETHER WITH THE PRE-MORPHISMS ON THE STACKS OF SITES AND THE COMPLETE MORPHISMS OF THE FIBER BUNDLES $\Phi_{(i)}$ UNDERLYING THE TOPOS.

REMARK: CLEARLY, AND AS EXPECTED, A TOPOS TURNS OUT TO BE A PARTICULAR INSTANCE OF OBJECTS Л AND WE NOTE WITH SATISFACTION THE CARTESIAN CLOSEDNESS OF OBJECTS Л WHEN VIEWED AS TOPOSES. IF Л IS A TOPOS, WE WILL WRITE $\text{Л} \underset{\text{DEF}}{=} \text{Л}^{\mathbf{E}}$

(NOTE THE ADJOINTED FORGETFUL FUNCTOR $\mathcal{J}_{\mathbf{E}}: \text{Л}^{\mathbf{E}} \Rightarrow \mathcal{E}$!)

NOW WE TURN TO THE TECHNICAL DETAILS OF THE PROPERTIES OF TOPOSES.

A RETRACING FUNCTION (RF) IS AN OBJECT WHICH ACTS ON A FIBER SUCH THAT THE (TERMINAL) OBJECTS $\mathbf{a}_i$, I.E. THE NOEMAS, ARE FIXED POINTS OF THE FUNCTION. FORMALLY, IF $\mathbf{r}$ DESIGNATES THE RF WE HAVE

$$\left|\begin{array}{l} \mathbf{r}\left(\left|\mathbf{a}_i\right|^{k+1}\right) \leqq \mathbf{r}\left(\left|\mathbf{a}_i\right|^{k}\right) \\ \\ \mathbf{r}(\mathbf{a}_i) = \varnothing \end{array}\right.$$

THE SET OF RETRACING FUNCTIONS OF A FIBER BUNDLE Φ IS CALLED THE RETRACT OF THE STACK GENERATED BY THE COLLECTION OF SITES FOR WHICH FIBRATIONS ARE DEFINED.

NEXT, A PULL-BACK (*pb*) IS THE "STACK" OF RETRACTS OF A TOPOS SUCH THAT THE FIXED POINTS OF THE RETRACTS (GENERATED BY THE RETRACING FUNCTIONS) COLLAPSE ON THE INITIAL NOEMA $\mathbf{a}_0$, WHICH THEN IS A UNIVERSAL TERMINAL OBJECT Л^{0}. THE PULL-BACK FUNCTION CAN BE DESCRIBED AS THAT FUNCTION WHICH TAKES A TOPOS $\text{Л}^{\mathbf{E}}$ AS ARGUMENT (ITS DOMAIN) AND YIELDS AS VALUE THE INITIAL NOEMA, $\mathbf{a}_0$. - CHARACTERISTICALLY, A PULL-BACK REVERSES ALL ARROWS IN A TOPOS $\text{Л}^{\mathbf{E}}$, I.E. IT IS LOCALLY CONTRAVARIANT.

THE DUAL CONCEPT OF A PULL-BACK FUNCTION *pb* IS THE NOTION OF A

Push-Out function po with interchanged domain and co-domain. Characteristically, then, the Push-Out takes as argument the Object $\mathbf{a}_o$ and yields as value the Topos $\boldsymbol{\Pi}^{\mathbf{E}}$. Intuitionistically, this will mean that the Push-Out provides us with a (derivational) Method for generating $\boldsymbol{\Pi}^{\mathbf{E}}$ given any presentation of $\mathbf{a}_o$. (A po determines a Spread).

Remark: A somewhat paradoxical situation arises from the fact that the dual nature of Push-Outs requires that their Fixed Points must also collapse. But the Fixed Point of a Push-Out is a non-empty set of Toposes (!), or, what amounts to the same, it collapses the class of all Toposes $\boldsymbol{\Pi}^{\mathbf{E}}$, on the singular Topos $\boldsymbol{\Pi}^{\mathbf{E}}$. Unfortunately, $\boldsymbol{\Pi}^{\mathbf{E}}$ is not unique upon this definition. But that may not necessarily be a problem, since every Topos $\boldsymbol{\Pi}^{\mathbf{E}}$ has a Pull-Back associated with it, and the Pull-Back yields a unique image for each Topos, viz. the Universal Terminal Object $\mathbf{a}_o = \boldsymbol{\Pi}^{o}$. Thus the paradox disappears if the Doctrine of Clubs over the clan of Toposes restricts proliferations of Fibrations that lead out of the Doctrine proper, i.e. the Doctrine advocates the Axioms of Irreproachability and Incontestability at every application of the Law of Sufficient Reason.

By a Sheaf we mean a Lattice of Toposes. Simplified, but not misleadingly, a Sheaf can be described as a "higher-order" Topos concept where quantification now is permitted over the entire class of Toposes. Thus, Sheaves might be regarded as many-sorted ramified type-structures over the universe of Toposes. As such, we should expect them to Reflect certain lower-order properties, i.e. properties associated with Toposes (such as CCC).

Consider any Rank Function appropriate for a Sheaf. Then the Sheaf is said to be normal if its Rank exceeds the Rank of the Toposes belonging to it. Formally, we mean that a Sheaf forms, in a certain sense, the Supremum of the Ranks of its Toposes. On the other hand, if the Rank of the Sheaf forms the Infimum of the Ranks of its Toposes, it will be said to be regressive. We can express this situation by noting that each Topos $\text{Л}^{\mathbf{E}}$ in a regressive Sheaf $\text{Л}^{\mathbf{E}^{\text{Л}}}$ has Rank > 1, and for most cases the Rank of a regressive Sheaf will be equal to 1.

It is desirable to adhere to Doctrines that avoid regressive Sheaves as they tend to be too similar to conventional(istic) art objects and their fallacious Modalities. Clubs that are unable to miss out on regressive structures are delegated to those entertaining impeachable logics.

The notion of Rank for a (normal) Sheaf Reflects down to its associated Toposes, as we noted above. The Reflexion is effected by a Natural Transformation (actually, a Galois Connection) $\rightsquigarrow$ preserving global properties of the Sheaf at its local regions occupied by Toposes and taking as image a prefabricated CCC Object $\text{Л}^{\mathbf{E}}$.

Recall that the Objects of the Doctrinal Category $\mathcal{E}$, i.e. the denotational connections between signs of type $[\mathbf{a}_i]^{k}$ or $\sqcup([\mathbf{a}_{i_1}]^{k_1}, \ldots\ldots, [\mathbf{a}_{i_n}]^{k_m})$ consist of the points of indications connected with the local Arrow or Pre-Morphism appear-

ING IN SOME FIBER FOR A SITE BELONGING TO THE TOPOS $\mathcal{E}$. THE RANK OR DEPTH OF ANY SUCH SIGN IS GIVEN BY ITS ASSOCIATED INDEX, I.E. $i+k$ OR $i+k+\frac{n+m}{2}$, AS THE CASE MAY BE. IN OTHER WORDS, THE DEPTH OF A SIGN $\left[\mathbf{a}_i\right]^k$ OR $\sqcup(\left[\mathbf{a}_{i_1}\right]^{k_1}, \ldots\ldots, \left[\mathbf{a}_{i_n}\right]^{k_m})$ EQUALS THE NUMBER OF ARROWS OR PRE-MORPHISMS OF THE UNDERLYING BUNDLE OR SINGLE FIBER THAT DEFINES THE CORRESPONDING INDICATIONS.

THE DEPTH OF AN INDICATION, IN TURN, IS GENERALLY EQUAL TO THE INVERSE OF THE RANK OF THE SITE OR STACK THAT ENVELOPES THE INDICATION. ANALOGOUSLY, THE DEPTH OF A TOPOS IS IN GENERAL EQUAL TO THE INVERSE OF THE RANK OF ITS SHEAF AND SO ON FOR THE REMAINING STRUCTURES (FIBER BUNDLES, pb'S, pc'S, SHEAVES, ETC.).

IN THE OTHER DIRECTION WE MAY REFLECT UPWARDS OVER THE ENTIRE UNIVERSE OF SHEAVES. AS A FIRST STAGE OF THIS REFLEXION, WE ARRIVE AT A PRE-COSMOI, WHILE AT A LATER STAGE THE APPEARANCE OF A COSMOI OF COLLECTIONS OF SETS OF SHEAVES BECOMES POSSIBLE.

FOR COSMOIS, WE MAY DEFINE LEFT AND RIGHT ADJOINTS AS A UNIVERSAL PROPERTY OF EVERY COSMOI. THAT MEANS IN PARTICULAR THAT EVERY COSMOI IS SYMMETRICALLY SELF-REFLEXIVE WITH RESPECT TO ADJOINTNESS.

ADJOINTS ASSOCIATED WITH SHEAVES AND TOPOSES ARE NOT NECESSARILY SYMMETRIC, BUT NEVERTHELESS DEFINED IN LEFT OR RIGHT FORM FOR EVERY SHEAF AND TOPOS. IF BOTH FORMS ARE PERMITTED BY THE DOCTRINE ON A SHEAF OR TOPOS, THE UNIVERSAL PROPERTY IS RECOVERED. SUCH SITUATIONS ARE REFERRED TO AS DOCTRINAL CLUB CONSPIRATIONS - DCC'S - AND THEIR OBJECTS ARE THE ADJOINTED SHEAVES AND TOPOSES.

In this way we can continue to Reflect upwards beyond the Cosmois towards more and more comprehensive Units and there seems to be no conceivable end to the iterative applications of the <u>Reflexion Principle</u>. On the other hand, due to the normality of our Sheaves, there is no corresponding infinite regress or descent downwards, since all structures below the Sheaves collapse ultimately on the void indication $\mathbf{a}_{\sigma}$ i.e. the Universal Terminal Object $\underset{\sim}{\Pi}^{\circ}$. This property will henceforth be referred to as the well-foundedness of the structure in question. On the contrary, regressive Sheaves or Toposes can never be well-founded and the same holds for their (inadmissible) Doctrines.

However, recalling Brouwer's insight concerning wasteful proliferations of Units, we shall stop at the Cosmois, for the time being, and leave to the interested reader to work out the details of the next stage, for which we on Doctrinal grounds have withdrawn every preassigned name. It occurs to us that this next stage is likely to behave more like a Black Hole, rather than, say, Continuous Plasmas, while the Pull-Back diagrams of Cosmois would be much more rewarding objects of study.

APPENDIX 1

TH(Σ) AND THE STRUCTURE OF MIND

BY A CREATIVE SUBJECT Σ WE SHALL NOW MEAN AN IDEALIZED COLLECTION OF STREAMS OF CONSCIOUSNESS , ξ_1 , ξ_2 , ..., ξ_k , ...

BY CONSCIOUSNESS WE MEAN ANY NON-EMPTY SET Ξ OF STREAMS ξ_{k_i} .

THE BASIS OF A CONSCIOUSNESS Ξ IS A SET Λ OF POTENTIALS λ_1 , λ_2, ..., λ_m , ... , DETERMINING THE RANGE OF THE CORRESPONDING STREAMS $\xi_k \in \Xi$. TYPICALLY, A POTENTIAL λ_{m_i} INITIATES AND SUPPORTS THE DEVELOPMENT OF A STREAM ξ_{k_j} . THE PROPERTIES OF λ_m ARE DESCRIBED IN TERMS OF THE CURRENTS ζ_{n_i} THAT λ_m MAY GIVE RISE TO ALONG ξ_{k_j} AND WHOSE TOTALITY WILL BE DESIGNATED Z.

CLEARLY $\Theta\{\Xi \sqcup \Lambda \sqcup Z\}$ DESCRIBES THE GENERATING FIELD OF THE TACTICS OF ATTENTION CONDUCTING ACTIVITIES OF Σ AT DIFFERENT STAGES OF HIS CONSCIOUSNESS DEVELOPMENT.

BY AWARENESS WE SHALL MEAN THE CONSCIOUSNESS OF ARROWS $\alpha_i \longrightarrow \alpha_{i+1}$ OCCURRING ALONG THE STREAMS ξ_{k_i} . BY $\alpha_i \longrightarrow \alpha_{i+1}$ WE ESTABLISH, AS USUAL, THE DENOTATIONAL CONNECTIONS BETWEEN ALL VALUES OF α_{i+1} THAT ARE VALUES OF α_i .

ARROWS PARTICIPATE IN CONCEPT FORMATION PROCESSES. THE KNOWLEDGE OF ARROWS IS SUBSUMED UNDER THOSE ARROW POTENTIALS ψ_ω WHICH FORCE ξ_k . THIS FORCING RELATION BRINGS INTO ATTENTION THE DENOTATIONAL CONNECTIONS WHICH SPRING FROM THE FIBRATION OF THE UNDERLYING CONCEPT FORMATION PROCESS. WHEN ξ_k BELONGS TO SOME Σ , THE STREAM ξ_k DESIGNATES ANY SUSTAINED FEELING

OF AWARENESS DIRECTED TOWARDS THE FIXED POINTS OF SOME MENTAL PHENOMENON WITHIN THE REALM OF Σ'S CONSCIOUSNESS. THE CORRESPONDING ARROW POTENTIALS ψ_{κ} ARE REFERRED TO AS THE NOEMAS Π' OF ξ_{κ}. FINALLY, THE MIND OF Σ IS DESIGNATED $\Xi \sqcup \Lambda \sqcup Z \sqcup \Psi$

THE CONSTRUCTION OF THE (INFINITE) CLASS $\Xi \sqcup \Lambda \sqcup Z \sqcup \Psi$ IS BASED ON TWO CONSTRUCTION PRINCIPLES AS FOLLOWS:

GIVEN A NOEMATIC BASIS Ψ FOR OBJECTS $\mathbf{a}_{00}$, $\mathbf{a}_{01}$, ..., $\mathbf{a}_{mn}$, ... ,(WHICH ACT AS OUR DESIGNATED ELEMENTS), A SIMPLEX σ OBTAINS IFF ONE OF THE FOLLOWING HOLDS:

1) $\sigma = |\mathbf{a}_{mn}|$, OR THE BRACKETING OPERATION

OR

2) $\sigma = \sqcup(|\mathbf{a}_{11}|', |\mathbf{a}_{01}|'' \ldots\ldots\ldots\ldots |\mathbf{a}_{mn}|'' \ldots$)

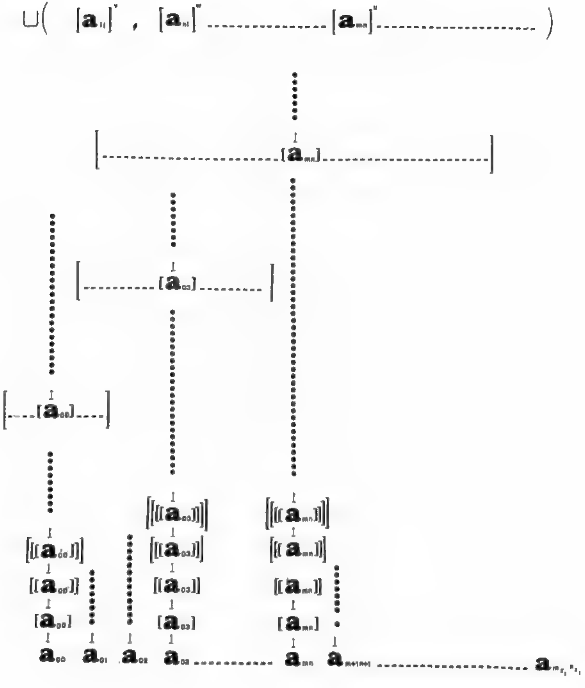
OR THE UNION OPERATION.

THE CLOSURE OF 1) AND 2) OVER $\Psi \sqsubset \Xi \sqcup \Lambda \sqcup Z \sqcup \Psi$ DESIGNATES THE UNIVERSAL MIND OF Σ RELATIVE THE BASIS Ψ. WE USE Σ^{Σ} TO SIGNIFY THE CORRESPONDING OBJECT.

THE DIAGRAM Δ OF Σ, $\Delta(\Sigma)$, INDICATES THE BASIS AND THE STATE OF GROWTH OF THE STREAMS OF CONSCIOUSNESS ξ_{κ} EMANATING FROM THE DESIGNATED ELEMENTS $\mathbf{a}_{00}$, $\mathbf{a}_{01}$, ..., $\mathbf{a}_{mn}$, ...

THE DIAGRAM FOR Σ OVER Ψ IS DISPLAYED ON THE FOLLOWING PAGE.

$\underline{\Delta(\Sigma)}$



APPENDIX 2

CARDINALS

IN

1— MODAL SPECTRA

CARDINALS IN 1-MODAL SPECTRA

By the CARDINAL of an Object K, Card(K), we mean the MEASURE of the size of K. There is a SMALLEST Cardinal, viz. Card(Ø), where (Ø) points to an EMPTY Object.

OPEN PROBLEM: Is there an Object K such that for every Object K′ different from K, Card(K) $>$ Card(K′), where $>$ designates the binary predicate "greater than"?

Let $\mathbf{K}$ be a CLASS of Cardinals. If there is a Cardinal $K \in \mathbf{K}$ such that for all Cardinals $K' \in \mathbf{K}$ Card(K) $>$ Card(K′), then $\mathbf{K}$ is BOUNDED ABOVE. If there is no such Cardinal, then $\mathbf{K}$ is UNBOUNDED and CLOSED in its COFINALITY. The COFINALITY of $\mathbf{K}$ is the class of Cardinals belonging to the ULTRAFILTER in the UPPER SEMI-LATTICE of $\mathbf{K}$ and which is closed under all regular Cardinal Operations Card$\{\Rightarrow\}$.

The invention of NEW Cardinals became one of the more fashionable pastimes during the 60's. Thus we have become acquainted with SACHS CARDINALS, SILVER CARDINALS, SOLOVAY CARDINALS, RAMSEY CARDINALS, ROWBOTTOM CARDINALS, just to mention a few. The invention of these Cardinals was often connected with the NEW MODELS FOR SET THEORY, as they became known in connection with Paul Cohen's famous FORCING CONSTRUCTIONS.(1)

This fashion has been well documented in the recent literature and its Objects constitute solid pieces of the Art of Con-

CEPTUAL THINKING, AND CERTAINLY A SPECIMEN OF A-ART. INDEPENDENTLY OF THESE CONTRIBUTIONS, WE NOW WISH TO INTRODUCE THE NOTION OF A SPREAD CARDINAL CARD($\underset{\sim}{\Sigma}$), WHERE $\underset{\sim}{\Sigma} = \langle \Sigma, \Sigma^{c} \rangle$ IS A SPREAD WITH Σ ITS SPREAD LAW OF FUNCTIONS FROM SEQUENCES $\mathbf{n}_0 \ldots\ldots\ldots, \mathbf{n}_j \ldots\ldots\ldots,$ INTO THE TRUTH SPACE $[0,1]$ AND Σ^{c} ITS COMPLEMENTARY LAW DEFINED ON THE FIELD OF Σ AND TAKING VALUES AMONG PREVIOUSLY CONSTRUCTED MATHEMATICAL ENTITIES.

CARD($\underset{\sim}{\Sigma}$) EXISTS SOLELY IN 1-MODAL SPECTRA FOR INTUITIONISTIC REASONINGS. BY THE PREFIX 1- WE EXPRESS AS USUAL MAXIMAL MODAL DEPTH, SO THAT THE NOTION CARD($\underset{\sim}{\Sigma}$) WILL DEPEND ON THE FOLLOWING BASIC OPERATION:

$$\Theta$$

THE CLOSURE OPERATOR OVER THE SPACE OF INTENSIONS $\mathcal{E}$.

FOR EXAMPLE, BY

$$\Theta[\pi_j]$$

WE MEAN THE CLOSURE OF INTENSION $\pi_j \in \mathcal{E}$, OR, IN OTHER WORDS, THE INTUITIONISTIC INTERIOR IMAGE OF THE NOEMA π_j IN THE MODALITY Θ, AS INDICATED ABOVE. THE III YIELDED BY AN APPLICATION OF Θ, AS ABOVE, IS SUCH THAT CARD(III) = CARD($\underset{\sim}{\Sigma}$) FOR THE SPREAD $\underset{\sim}{\Sigma}$ ASSOCIATED WITH THIS APPLICATION.

PREPARING FOR THE THEOREMS BELOW, WE INTRODUCE THE DISTINCTION BETWEEN SMALL CARDINALS AND LARGE CARDINALS. BY SMALL CARDINALS WE MEAN OBJECTS K SUCH THAT CARD(K) AND K IS CONSTRUCTIBLE WITHIN GÖDEL'S CONSTRUCTIBLE UNIVERSE L OR IN ITS JENSEN REFINEMENT J. WE THINK OF THE "REAL" (CANTORIAN) WORLD V AS CONTAINING, AT LEAST, THE HIERARCHIES L OR J.

IF WE RESTRICT OUR ATTENTION JUST TO THE CONSTRUCTIBLE LEVELS IN V, WE SHALL WRITE $V = L$ OR $V = J$. (THE FORMER EXPRESSION IS ALSO KNOWN AS THE AXIOM OF CONSTRUCTIBILITY, FIRST FORMULATED BY GÖDEL, 1938).

IF $\mathbf{K}$ IS AN OBJECT AND CARD($\mathbf{K}$) AND $\mathbf{K}$ IS NOT CONSTRUCTIBLE IN L OR J, THEN CARD($\mathbf{K}$) IS A LARGE CARDINAL. IN OUR LANGUAGE, THE MOST OBVIOUS LARGE CARDINAL IS, OF COURSE, CARD($\mathcal{E}$).

BY WHAT WAS SAID ABOUT THE RELATION $\Theta|\pi_{,}|$ = III ABOVE, WE MAY NOW STATE

THEOREM 1: E V E R Y   S P R E A D   C A R D I N A L CARD($\underset{\sim}{\Sigma}$) I S   C O N T A I N E D   I N T H E   L A R G E   C A R D I N A L $\mathcal{E}$

RE: THE OPEN PROBLEM ABOVE. ONE MIGHT THINK $\mathcal{E}$ IS SUCH THAT FOR ALL K IN OUR UNIVERSE, CARD($\mathcal{E}$) $>$ CARD(K). TRIVIALLY, THIS HOLDS IN EVERY 0-MODAL SPECTRUM DUE TO THE COMPLETE COFINAL CHARACTER OF EVERY K IN 0-MODAL SPECTRA. FOR WHICH OTHER PREFIXES IT MAY HOLD STILL REMAINS AN OPEN PROBLEM!

ALTHOUGH THE NOTION CARD MEASURES A SIZE OF K, WHICH WAS EXPRESSED BY CARD(K) ABOVE, IT MIGHT NOT ALWAYS BE THE CASE THAT EVERY K IS MEASURABLE ALTHOUGH WE HAVE CARD(K). HENCE WE MAY INTRODUCE ANOTHER NOTION ABOUT CARDINALS, VIZ. THE MEASURABLE CARDINALS. FOR A COMPLETE DISTINCTION, WE SHALL ALSO INTRODUCE NON-MEASURABLE CARDINALS.

BY A MEASURE μ ON K, WE SHALL UNDERSTAND THE MAP $\mu : 2^K \rightarrow \{0,1\}$, WHERE 2^K IS THE TOTAL PARTITION OF THE PARTS AND SUBPARTS OF K COLLECTED TOGETHER (I.E., 2^K

IS THE POWER SET OF K) AND WHICH SATISFIES:

$$\left| \begin{array}{l} \mu(K) = 1 \\ \mu(K') = \sum \mu(K_j') \end{array} \right.$$

WHERE $K' \in 2^K$, $K' = K_1' \sqcup K_2' \sqcup \ldots\ldots\ldots \sqcup K_j'$ AND $K_1' \sqcap K_2' \sqcap \ldots\ldots\ldots \sqcap K_j' = 0$ (N. B.: WE USE $\sum$ AS AN ADDITIVE SIGN, WHILE $\sqcup$, $\sqcap$ ARE USED AS REGULAR UNIONS AND INTERSECTIONS RESPECTIVELY).

THEOREM 2: IF A SPREAD CARDINAL CARD(Σ) IS A LARGE CARDINAL, THEN IT HAS A NON-TRIVIAL TOTAL MEASURE $\mu = 1$

THE FOLLOWING THEOREM IS, THEN, RATHER OBVIOUS:

THEOREM 3: A SPREAD CARDINAL IS A MEASURABLE CARDINAL IFF[1] IT IS LARGE

[1] IF, AND ONLY IF

WE NOW STATE TWO THEOREMS FOR THE ADJOINTS $\square$ AND $\blacksquare$

THEOREM 4: $\square_1$ IS NON-MEASURABLE

PROOF: BY EXPERIENCE (AND CONSEQUENTLY, ALREADY BY DEFINITION)

THEOREM 5: [redacted] H A S   M E A S U R E   1 :

$$\mu (\text{[redacted]}) = |\pi^{\circ}|$$

PROOF: BY DEFINITION, EVERY 1-MODAL SPECTRUM HAS $1 \overset{\nabla}{=} |\pi^{\circ}|$ (∇ IS, OF COURSE, OUR PAST TENSE OPERATOR)

OF INTEREST IS ALSO:

THEOREM 6: F O R   A N Y [redacted] I S E L E M E N T A R I L Y   E Q U I V A L E N T W I T H $\mathcal{E}$:

$$\text{[redacted]} \approx \mathcal{E}$$

NOT SURPRISINGLY, THEN:

COROLLARY: CARD([redacted]) I S   A   L A R G E C A R D I N A L

AS WAS TO BE EXPECTED, [redacted] IS CONSEQUENTLY NOT IN L OR J . SO,

COROLLARY: [redacted] I S   N O T   C O N S T R U C T - I B L E   I N L .

WHERE WE RECALL THAT L STANDS FOR GODEL'S CONSTRUCTIBLE UNIVERSE.

FOR THE EXPERT, WE MAY ALSO MENTION:

<u>COROLLARY</u>: <u>BY CONSTRUCTING</u> [redacted] <u>IN</u> J , <u>WE OBTAIN A MIRACLE</u>

FOR ADJOINTS $\square \quad \boxed{\square}$ WE HAVE THE FOLLOWING:

<u>THEOREM 7</u>: $\square \quad \boxed{\square}$ <u>IS ONLY MEASURABLE BELOW</u> $[\pi^0]$ <u>IMPLIES</u>

CARD($\square \quad \boxed{\square}$) = CARD(Σ)

<u>AND, FURTHER, FOR NO</u> $j > 0$ <u>IS</u> $[\pi_j]$ <u>MEASURABLE</u>

THUS, WE HAVE THE RATHER SENSATIONAL

<u>THEOREM 8</u>: <u>IF</u> CARD($\square \quad \boxed{\square}$) = CARD(Σ),

<u>THEN</u> CARD($\square \quad \boxed{\square}$)

<u>IS THE LEAST MEASURABLE CARDINAL</u>

<u>PROOF</u>: EITHER BY FINE SENSES OR, ALTERNATIVELY, BY THE FOLLOWING CONSIDERATIONS.

PUT $\mu(|\boxed{\square}|) = \mu(|\square|) = 0$.

THEN WE MAY PUT

μ(CARD($\square \quad \boxed{\square}$)) = 1 = $[\pi^0]$

Ad (1). In Theorem 4 above, we stated the Non-Measurability of $\square_1$. By arrows, it follows that $\blacksquare$ also is Non-Measurable and in fact each $\blacksquare$, is Non-Measurable. This is one of the basic facts underlying the Fine Senses Lemma given in another part of this work.

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[1](cf. A.R.D. Mathias: The Surrealistic Landscape of Set Theory after Cohen. Dept of mathematics, UCLA. Underground, 1967)

APPENDIX 3

TOPOSES & ADJOINTS

4 ix

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17 x

76

11 am

|

9 pm

TOPOSES AND ADJOINTS

BY CHRISTER HENNIX

A SURVEY OF THE FORMATION OF ABSTRACT CONCEPTS
FROM CANTOR TO LAWVERE

MODERNA MUSEET, STOCKHOLM

4.9.1976 - 17.10.1976

I. INVISIBLE PROCESS PIÈCES

0. COMPOSITE CONTINUOUS INFINITARY WAVE-FORM ((ELECTRONIC SINE WAVES) - (QUOTED WORD OBJECT FROM LINCOS FOR LA MONTE YOUNG, 1969 -))

0^1. AROMATIC CHAINS #4 ((SUSANNA'S) SUPER SENSUALISM, COLLABORATION PIECE WITH SUSANNA KAE, 1973 -) ------- ------------------------ (ORGANIC MOLECULES IN AIR)

II. SET & SPREAD PIECES (TOPOLOGIES)

1. SHORT INFINITARY PROCESS ---------------- (ACRYLIC, 1973) (USING THE RESTRICTED TACTIC OF ATTENTION)

2. MODEL FOR SET THEORY, $\mathcal{E}$ ---------------- (ACRYLIC, 1973) (OBJECT IN THE CATEGORY OF ALL CATEGORIES)

3. Open Point Set of Measure 0 .... (Letraset, 1971)
(Continuously Variable Lawverian Set)

4. Straight Lines .... (Acrylic, 1971)

5*. Brouwer's Bar .... (Stainless steel, 1975)
(Answering a question of Walter De Maria by use of Brouwer's Bar Theorem)

6*. The Least Non-Measurable Cardinal .... (Stainless steel, 1973-)

7*. The Least Measurable Cardinal ..(Stainless steel, 1973-)

8*. Brackets .... (Stainless steel, 1970)

(*= Made 1976 by special agreement with Nord Verkstad AB, Molkom (Uddeholm))

III. Exercise Pieces .... (1976-)

9. Exercise #1 ....(Letraset)

10. Exercise #2 ....(Letraset)

11. Exercise #3 ....(Soot on aluminum)
(**2** - and **C** -spaces of Abstract Topologies)

IV. Toposes Computer Monitor Displays (Dia projections 1974-)

12. Topos #1

13. Topos #2

14. Topos #3
15. Topos #4
16. Topos #5
17. Topos #6
18. Topos #7
19. Topos #8
20. Topos #9
21. Topos #10
22. Topos #11
23. Topos #12
24. Topos #13
25. Topos #14

V. Language Pieces

26. (Fragments from) LINCOS (For Intergalactic Communications) - (Illuminated dias) (Infinitary Drawing, 1969)

VI. Abstract Nonsense Pieces (Typewriter & letraset)

27. Excerpts from Notes on Toposes and Adjoints (1969-76)

VII: Historical Department

28. Kleene's Slash .... (Letraset)

29. First Commutative Diagrams ... (Reproduction technique) (After Eilenberg - MacLane, 1942)

VIII: Appendix

30. Gödel's L .... (Reproduction technique)

31. Jensen's J .... (" ")